Software Model Checking: Theory and Practice

Lecture: *Specification Checking - LTL Model Checking*

Objectives

- To understand Büchi automata and their relationship to LTL
- To understand how Büchi acceptance search enables a general LTL model checking algorithm

Safety Checking

For safety properties we automated the "instrumentation" of checking for acceptance of a regular expression for a violation

This involved modifying the DFS algorithm to

- Calculate states of the property automaton
- Check to see whether an accept state is reached

We will apply the same basic strategy for LTL

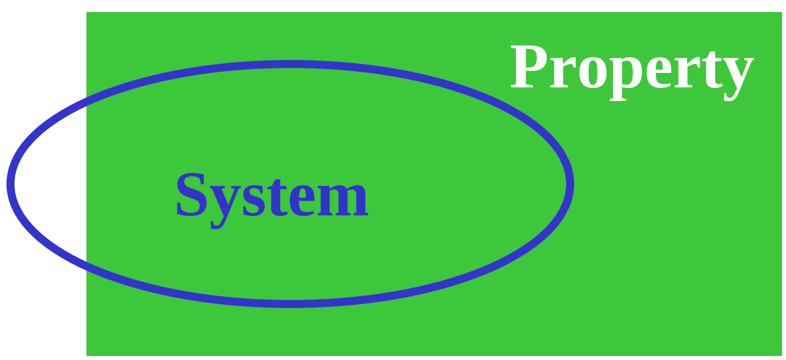
LTL Model Checking

From the semantics

- An LTL formula defines a set of (accepting) traces

We can

- Check for trace containment



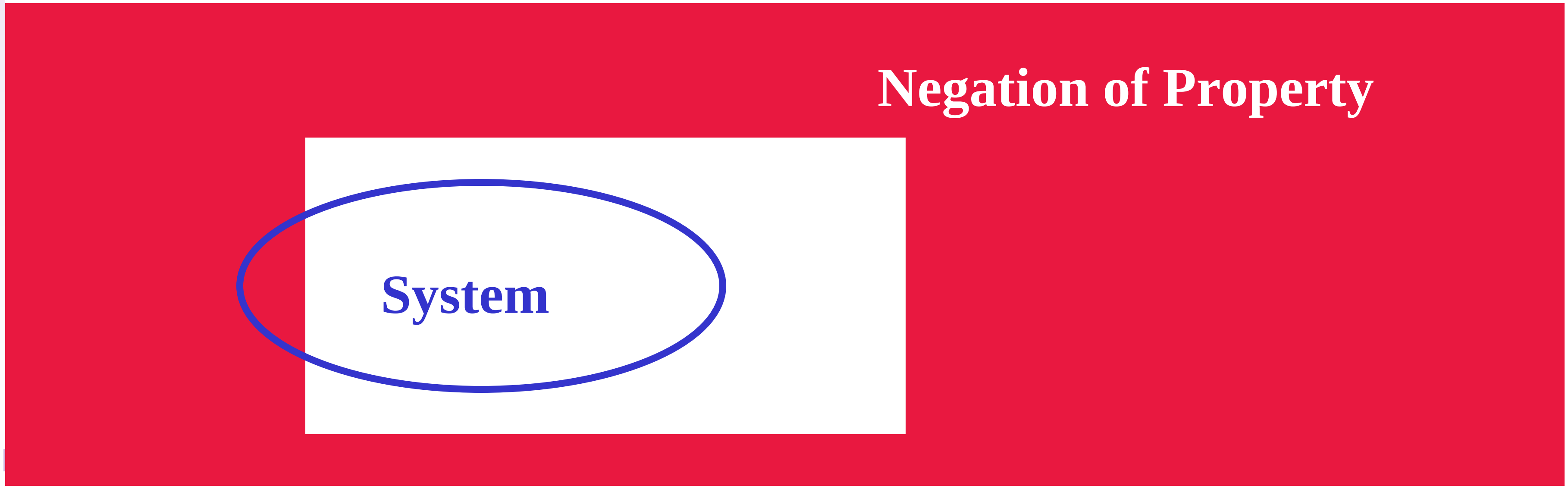
LTL Model Checking

From the semantics

- The negation of an LTL formula defines a set of (violating) traces

We can

- Check for non-empty language intersection



Emptiness Check

LTL is closed under complement

$$L(\phi) = \overline{L(\neg\phi)}$$

where the language of a formula defines a set of *infinite* traces

A Büchi automaton accepts a set of infinite traces

Büchi Automata

A Büchi automaton is a quadruple (R, I, δ, F)

S is a set of states

$I \subseteq R$ is a set of initial states

$\delta : R \rightarrow \mathrm{P}(R)$ is a transition relation

F is a set of accepting states

Unlike FSAs, Büchi automata are always non-deterministic

- set of initial states
- multiple transitions from a state

Büchi Automata

Automaton states are labeled with atomic propositions of the formula

$\lambda : R \rightarrow \mathrm{P}(A)$

- where A are the set of observables for the program
- λ(r) is the set of observables for a property state

Note that the meaning of the automata is defined via this mapping

- plays the role of alphabet in FSA

Example : Büchi Automaton

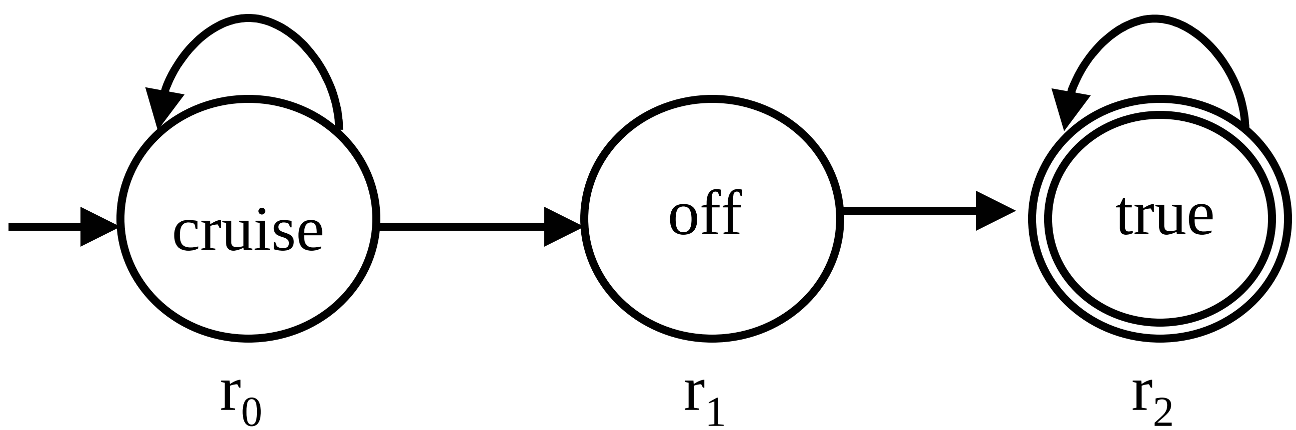
S = {r_0, r_1, r_2}

I = {r_0}

δ = {(r_0,{r_0,r_1}),(r_1,{r_2}),(r_2,{r_2})}

F = {r_2}

λ = {(r_0,cruire}),(r_1,{off}),(r_2,{ })}



Büchi Automata Semantics

An infinite trace

$\sigma = r_0, r_1, \ldots$

is accepted by a Büchi automaton iff

$r_0 \in I$ *starting in an initial state*

$\forall i \geq 0 : r_{i+1} \in \delta(r_i)$ trace corresponds to transition relation

$\forall i \geq 0 \ \exists j \geq i : r_j \in F$ *can reach a final state* from end of all prefixes

Büchi Trace Containment

Assume each system state (S) is labeled (Λ) with set of observables (A)

A Büchi automaton accepts a system trace

$$s_0, s_1, \ldots$$

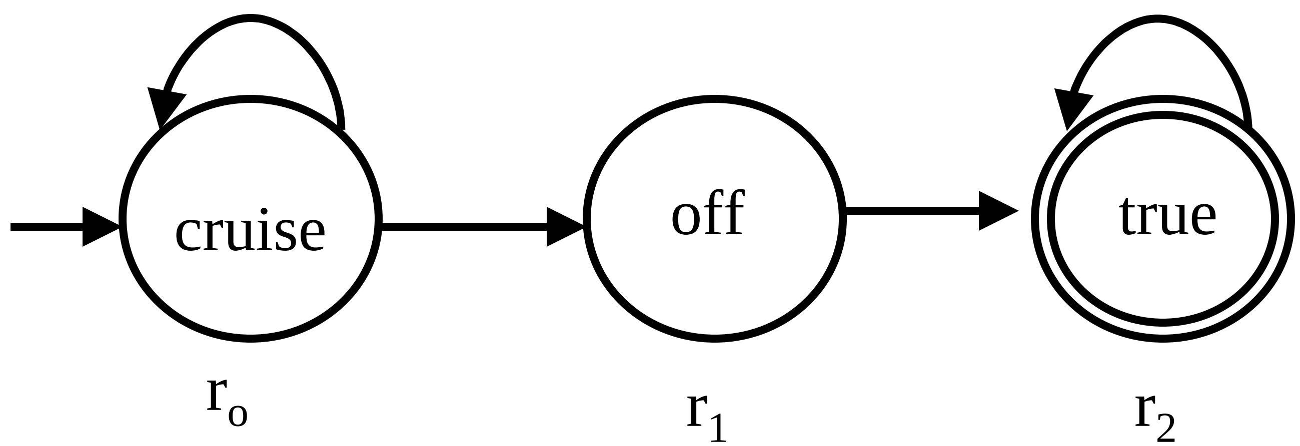
$$\exists r_0 \in I : \lambda(r_0) \in \Lambda(s_0)$$

$$\forall i \geq 0 \; \exists r_{i+1} \in \delta(r_i) : \lambda(r_{i+1}) \in \Lambda(s_{i+1})$$

$$\forall i \geq 0 \; \exists j \geq i : r_j \in F$$

Example : Büchi Automaton

cruise cruise off off accel accel cruise ...
cruise cruise accel cruise off accel ...



LTL and Büchi Automata

- Every LTL formula has a Büchi automaton that accepts its language (not vice versa)

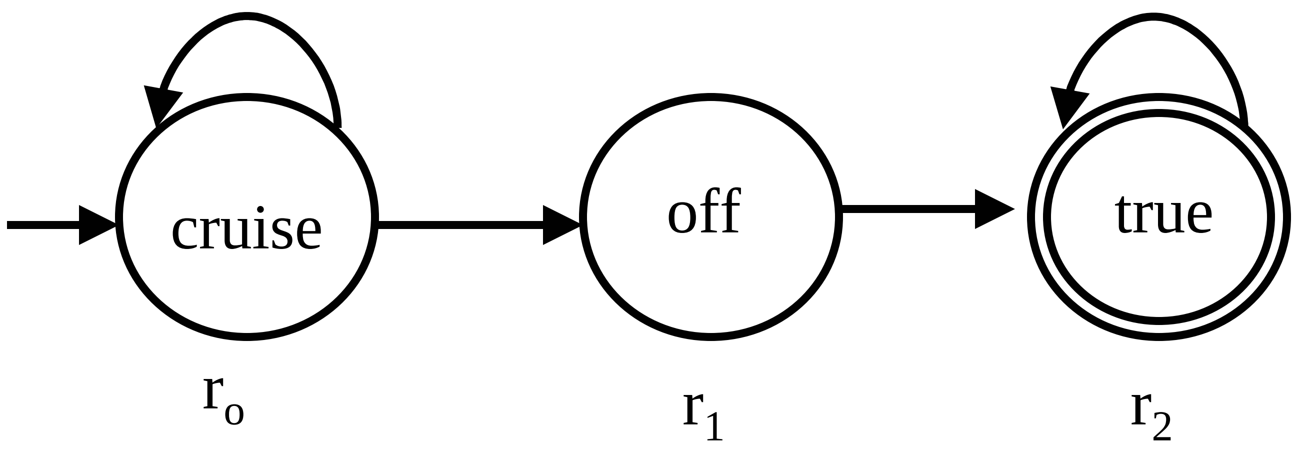
  $$L(\phi) \subseteq L(BA)$$

  $$L(\phi) \cap L(BA) \neq \emptyset$$

- Büchi automata cannot be determinized
  - i.e., there is no canonical deterministic automaton that accepts the same language
- Büchi automata are closed under the standard set operations

Example : Büchi Automaton

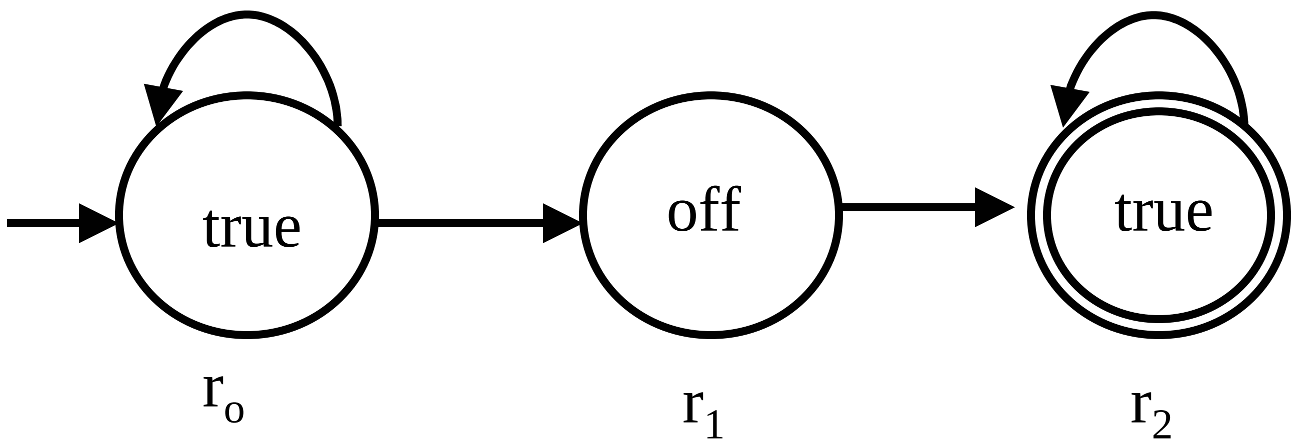
What LTL property does this correspond to?



cruise U off

Example : Büchi Automaton

What LTL property does this correspond to?



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LTL Model Checking

- Apply same strategy as before
  - Generate Büchi automaton for the negation of the LTL property
  - Explore state space of the product of the automaton and the system
  - Check for emptiness
- Violation are indicated by accepting traces
  - Look for cycles containing an accept state
  - Use nested depth-first search

LTL Model Checking

```
errors := {}
seen := {}
for each r ∈ / do
    seen := seen ∪ {(s0 , r)}
    stack := [(s0 , r)]
    DFS((s0 , r))
    pop(stack)
```

For each initial property state

initialize DFS data structures

perform search of initial product state

LTL Model Checking

```
DFS ((s,r))
    workSet := enabled (s )
    for each α ∈ workSet do
        s' := α (s )
        for each r' ∈ δ(r) do

            if λ(r') ∉ Λ(s') then
                if (s' ,r' ) ∉ seen then
                    seen := seen ∪ {(s' ,r' )}
                    push (stack, (s' ,r' ))
                    DFS((s',r'))
                    if r' ∈ A then
                        seen' := {(s' ,r')}
                        stack' := [(s' ,r')]
                        NDFS((s',r'),(s' ,r'))
                        pop(stack' )
        pop(stack)
end DFS
```

For each transition

check if state labels match

Only launch a cycle search from property accept states

LTL Model Checking

```
NDFS ((s,r), seed)
    workSet := enabled (s )
    for each α ∈ workSet do
        s' := α (s )
        for each r' ∈ δ(r) do

            if λ(r') ∉ Λ(s') then
                if (s',r') ℑ ∫⌉⌉⌈ then
                    errors := errors U {(stack,stack')}
                    continue
                if (s',r') ∉ seen' then
                    seen' := seen' ∪ {(s',r')}
                    push (stack', (s',r'))
                    NDFS((s',r'), seed)
                    pop(stack')
end NDFS
```

For each transition

check if state labels match

Same logic as for

progress checking

Fairness

- Liveness states that the system should eventually do something
  - Often times in real systems threads rely on a schedule to give them a chance to run
  - Abstracting scheduling to non-deterministic choice introduces severe approximation
- There are many forms of fairness
  - The intuition is that we restrict the systems behaviors to only those on which each process gets a chance to execute

Fairness in LTL

- LTL is expressive enough to state fairness properties directly
  - []<> (Phil1.eating || Phil2.eating)
  - ([]<>Phil1.eating) && ([]<>Phil2.eating)
- Fairness formula can be used to *filter* the behaviors that are checked as follows
  - Fairness -> Property
  - If not Fairness then whole thing is true
  - Property checked only when Fairness holds