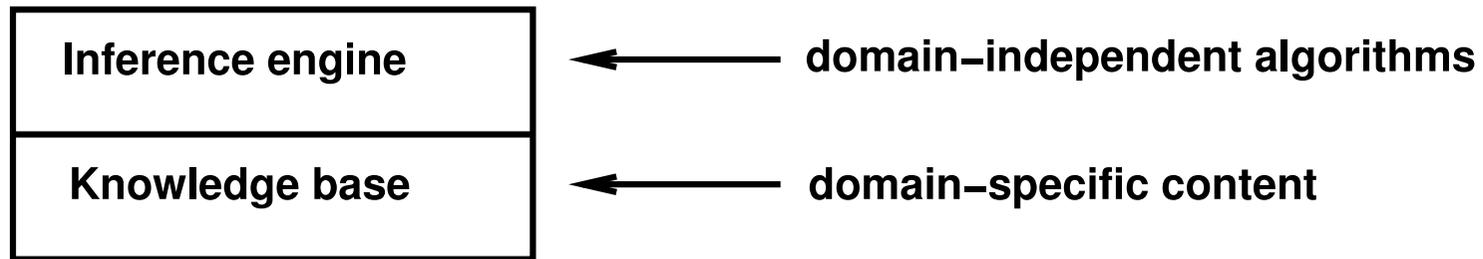


**Einschub:
Aussagenlogik**
(in Englisch)

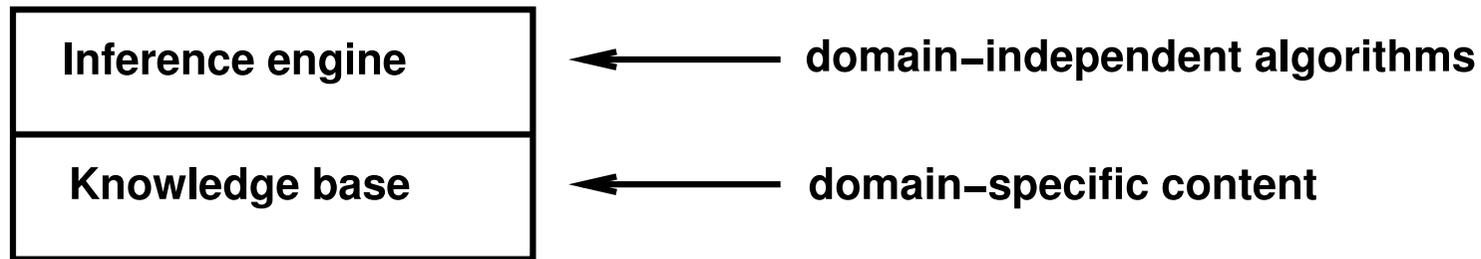
Knowledge bases



Knowledge base

Set of **sentences** in a **formal** language

Knowledge bases



Knowledge base

Set of **sentences** in a **formal** language

Declarative approach to building an agent

Tell it what it needs to know

Then it can ask itself what to do—answers follow from the knowledge base

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square

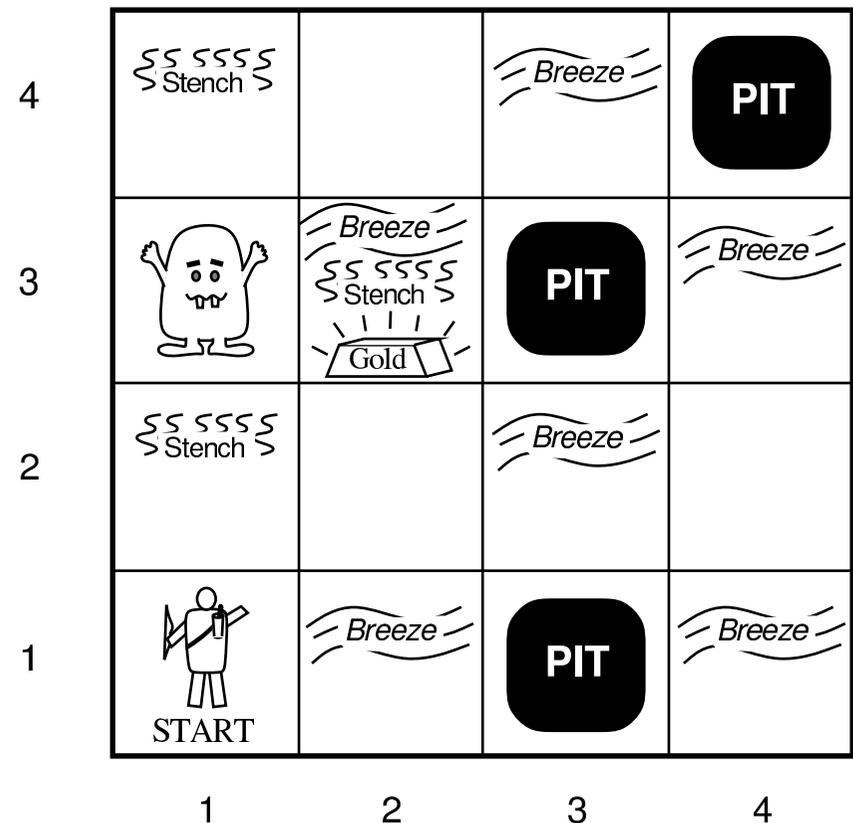
Actuators

Left turn, Right turn,

Forward, Grab, Release, Shoot

Sensors

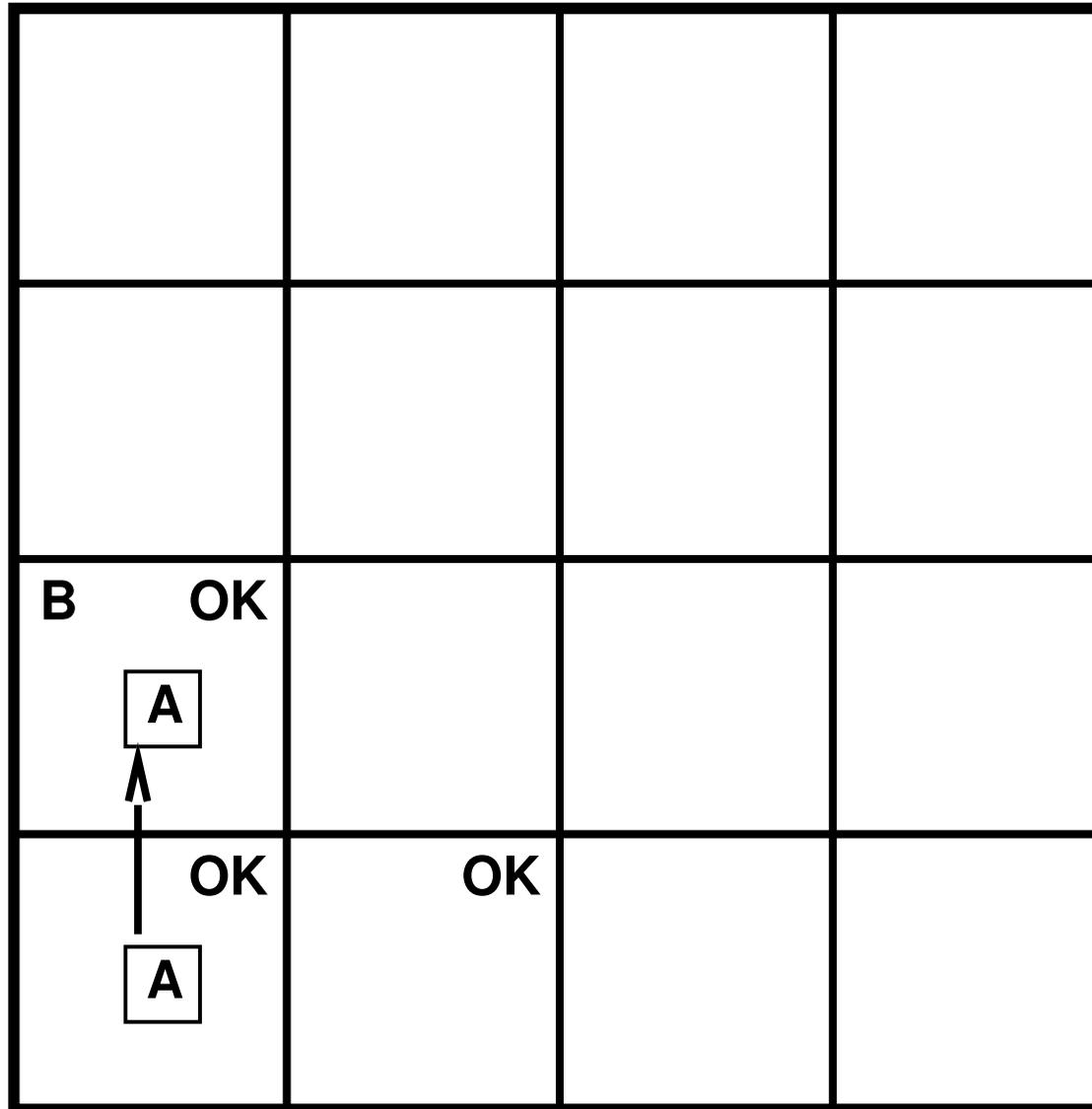
Breeze, Glitter, Smell



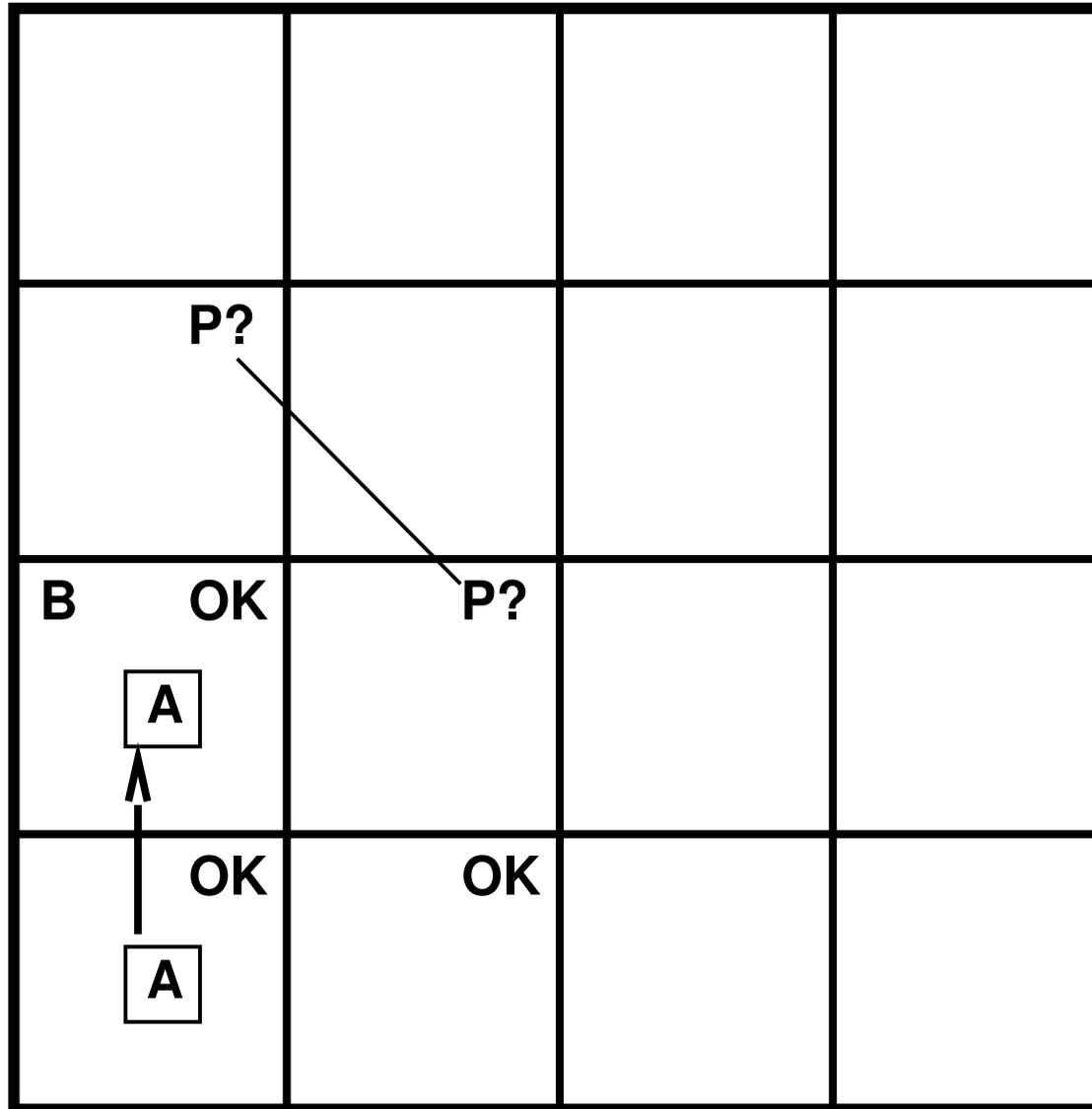
Exploring a Wumpus World

OK			
OK A	OK		

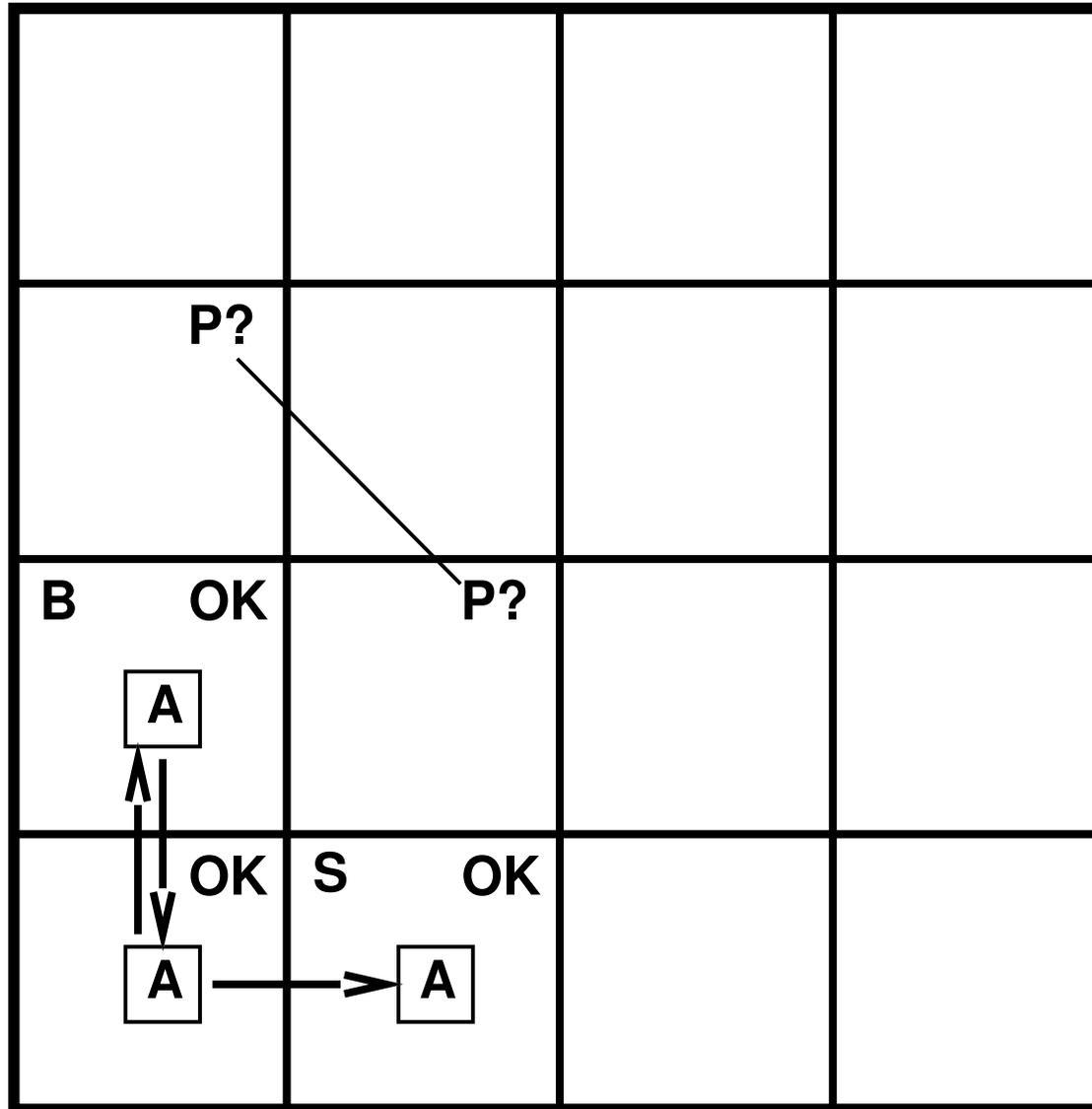
Exploring a Wumpus World



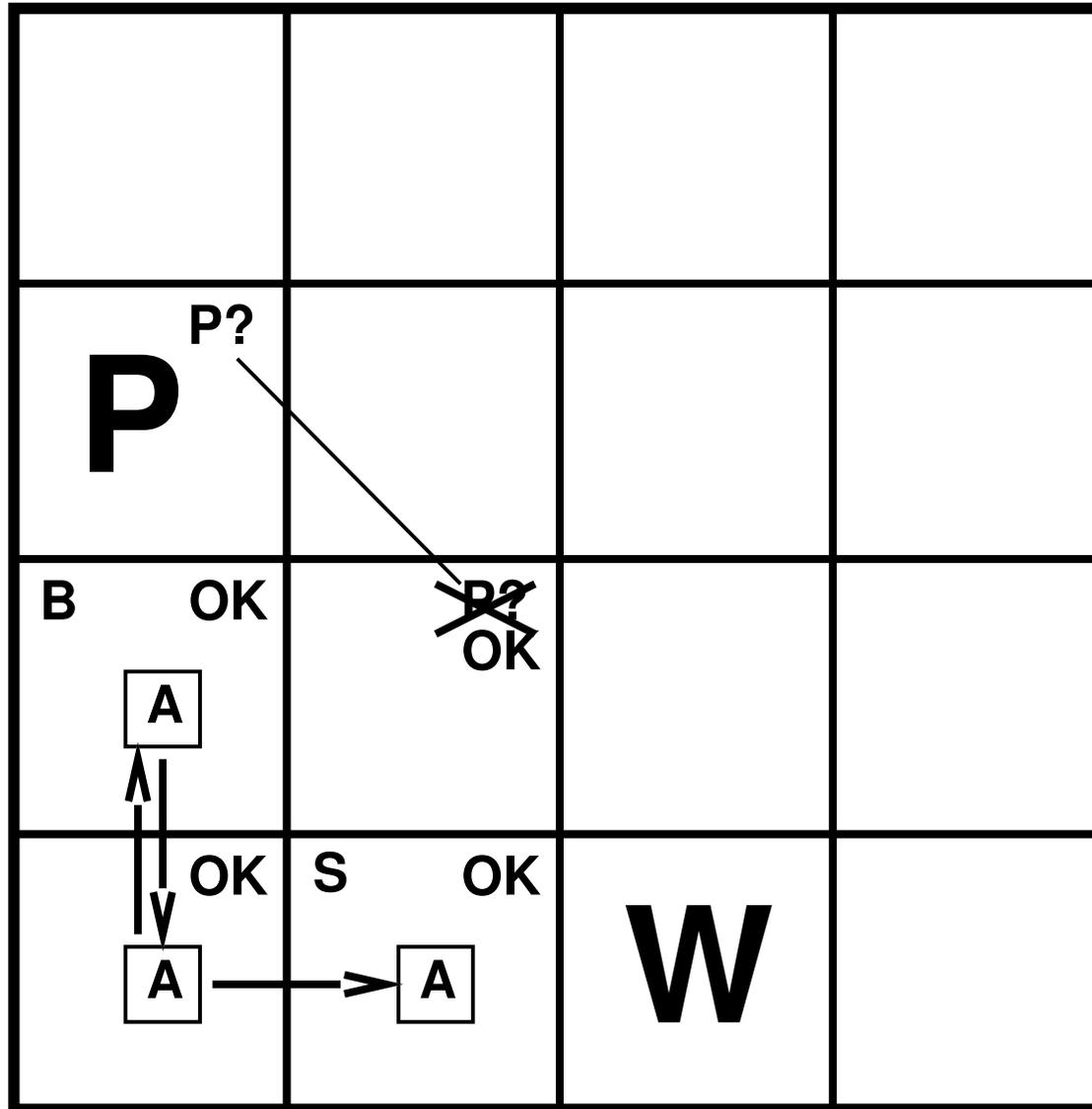
Exploring a Wumpus World



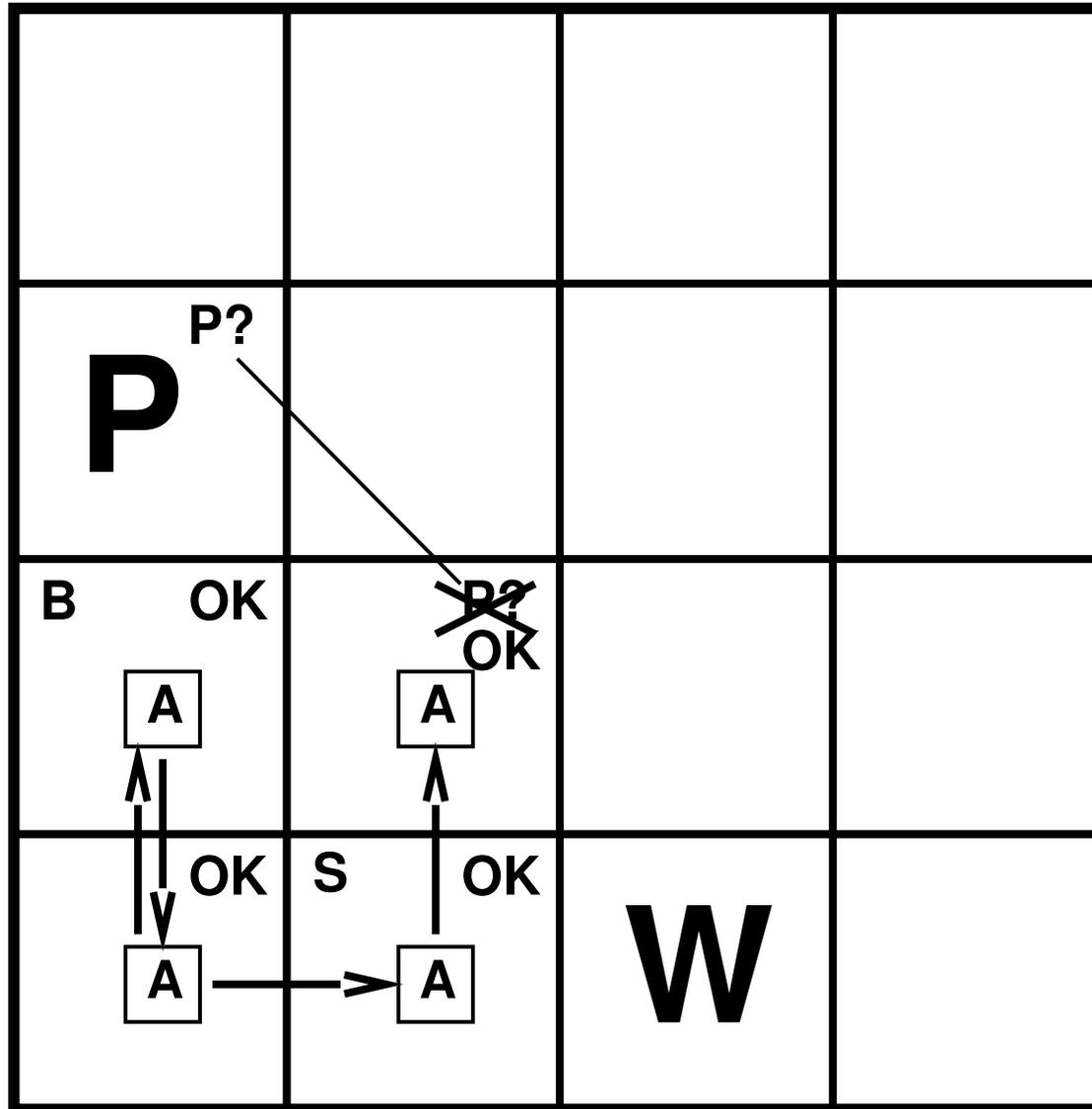
Exploring a Wumpus World



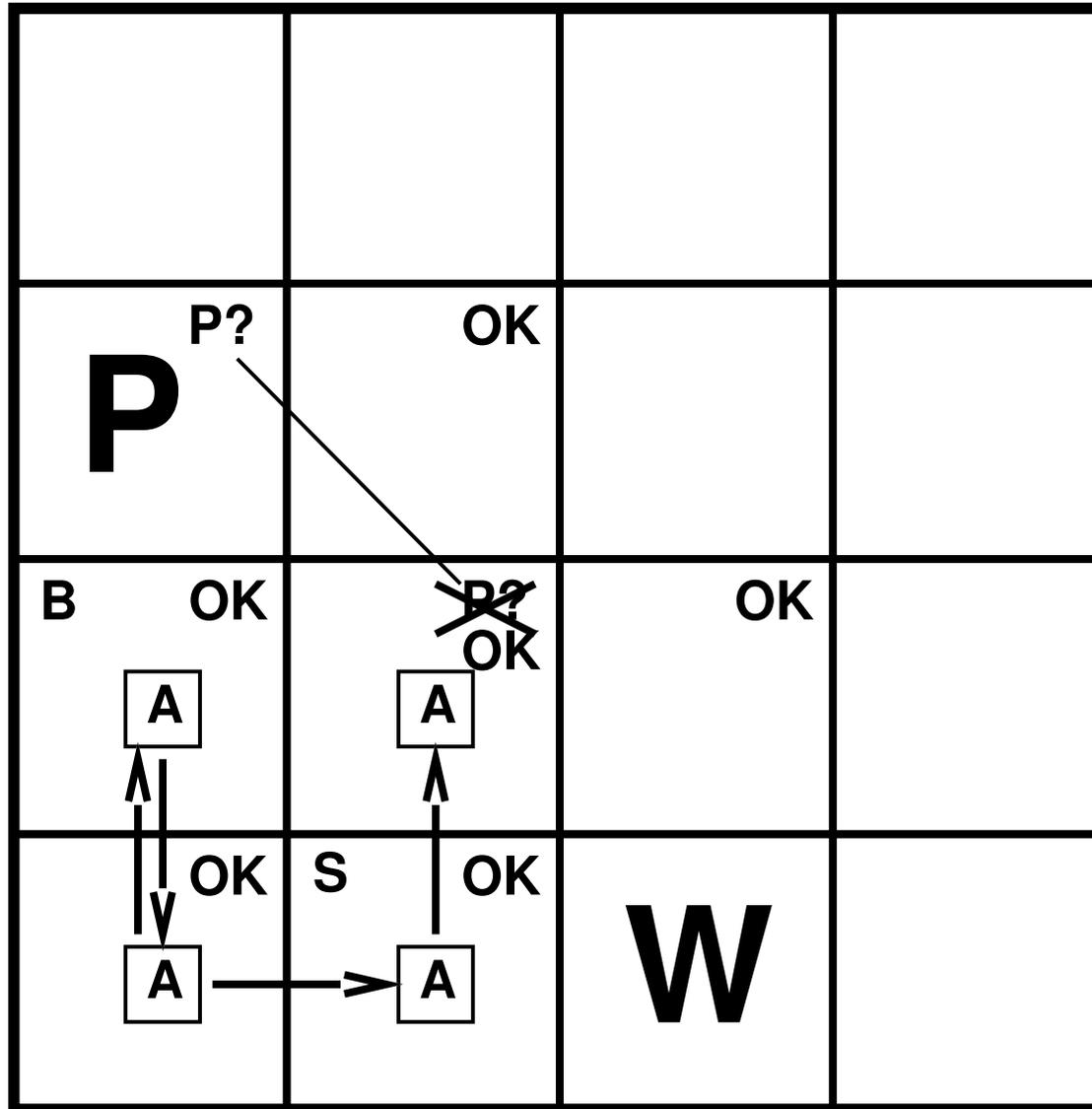
Exploring a Wumpus World



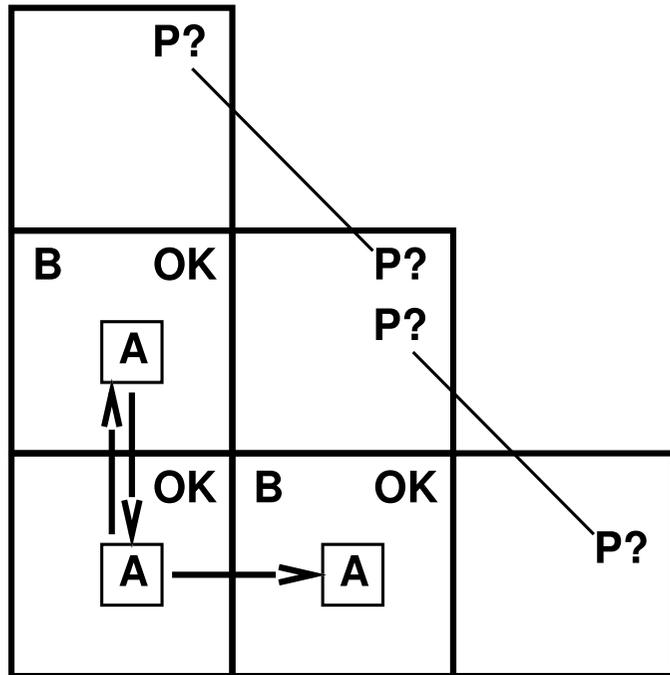
Exploring a Wumpus World



Exploring a Wumpus World



Problematic Situations

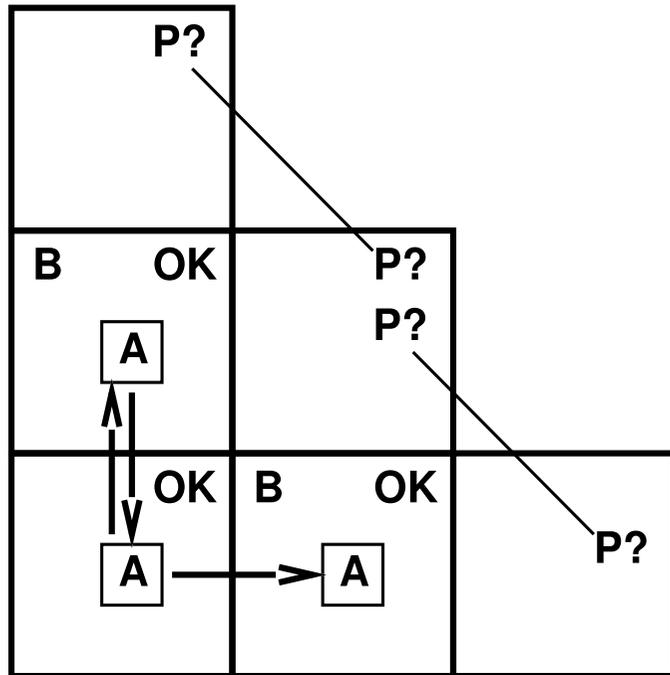


Problem

Breeze in (1,2) and (2,1)

⇒ no safe actions

Problematic Situations



Problem

Breeze in (1,2) and (2,1)

⇒ no safe actions

Possible solution

Assuming pits uniformly distributed:

(2,2) has pit with probability 0.86

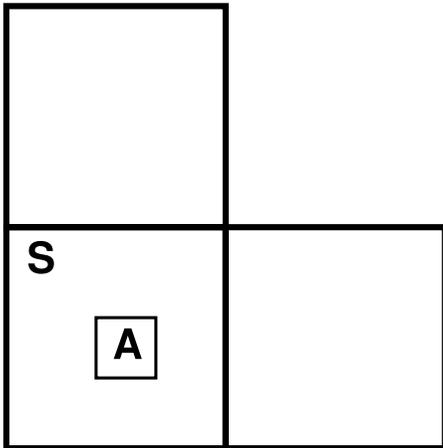
(1,3) and (3,1) have pit with probab. 0.31

Problematic Situations

Problem

Smell in (1,1)

⇒ no safe actions

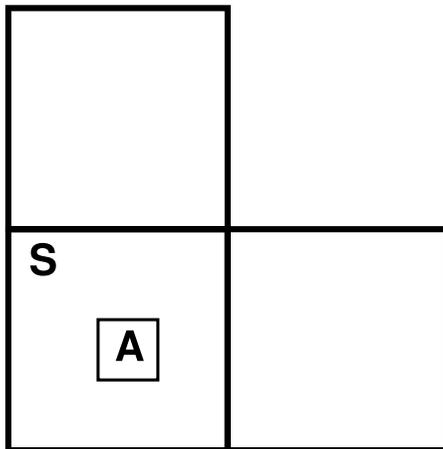


Problematic Situations

Problem

Smell in (1,1)

⇒ no safe actions



Possible solution

Strategy of coercion:

shoot straight ahead

wumpus was there ⇒ dead ⇒ safe

wumpus wasn't there ⇒ safe

Logic in General

Logics

Formal languages for representing information,
such that conclusions can be drawn

Syntax

Defines the sentences in the language

Semantics

Defines the “meaning” of sentences;
i.e., defines **truth** of a sentence in a world

Example: Language of Arithmetic

Syntax

$x + 2 \geq y$ **is a sentence**

$x^2 + y >$ **is not a sentence**

Example: Language of Arithmetic

Syntax

$x + 2 \geq y$ **is a sentence**

$x^2 + y >$ **is not a sentence**

Semantics

$x + 2 \geq y$ **is true iff the number $x + 2$ is no less than the number y**

$x + 2 \geq y$ **is true in a world where $x = 7, y = 1$**

$x + 2 \geq y$ **is false in a world where $x = 0, y = 6$**

Entailment

Definition

**Knowledge base KB entails sentence α
if and only if
 α is true in all worlds where KB is true**

Notation

$$KB \models \alpha$$

Entailment

Definition

Knowledge base KB entails sentence α
if and only if
 α is true in all worlds where KB is true

Notation

$$KB \models \alpha$$

Note

Entailment is a relationship between sentences (i.e., **syntax**)
that is based on **semantics**

Entailment

Example

The KB containing “the shirt is green” and “the shirt is striped” entails “the shirt is green or the shirt is striped”

Example

$x + y = 4$ entails $4 = x + y$

Models

Intuition

**Models are formally structured worlds,
with respect to which truth can be evaluated**

Models

Intuition

Models are formally structured worlds,
with respect to which truth can be evaluated

Definition

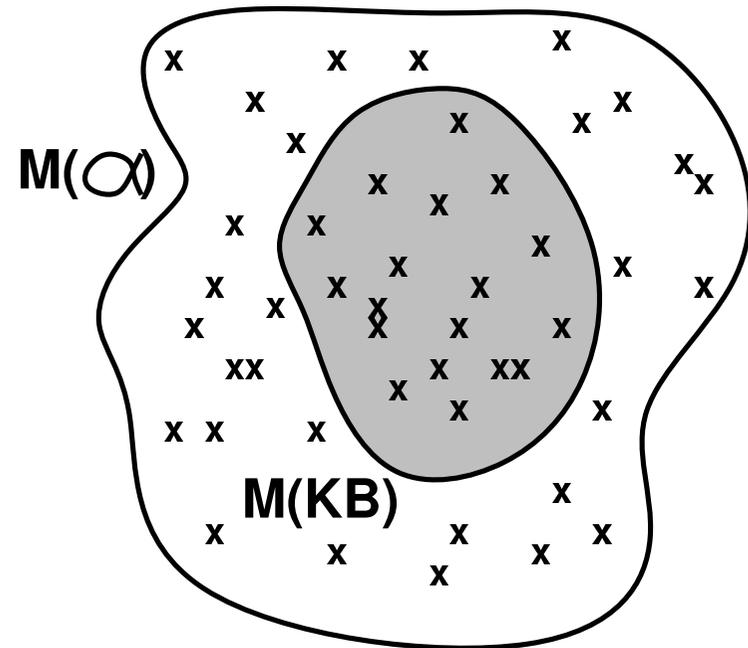
m **is a model of** a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

Note

$KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

Models: Example



KB = The shirt is green and striped

α = The shirt is green

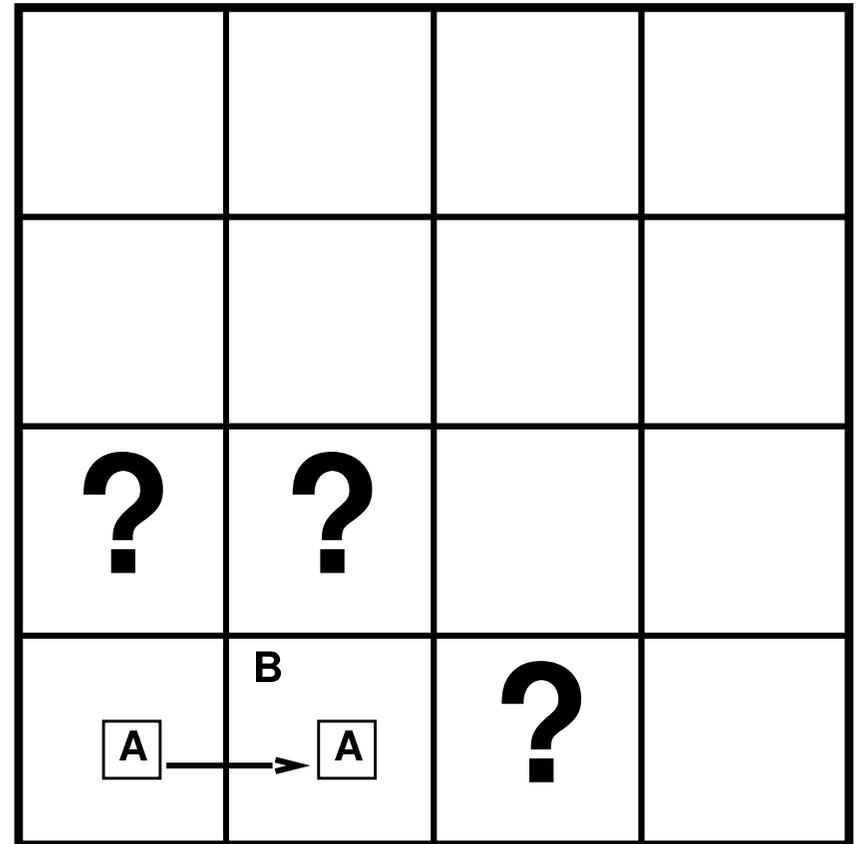
Entailment in the Wumpus World

Situation after

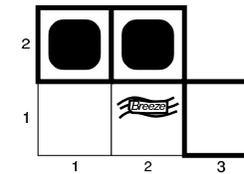
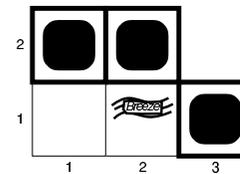
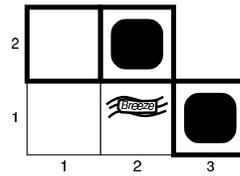
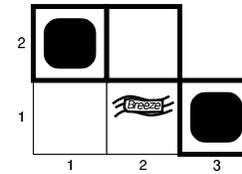
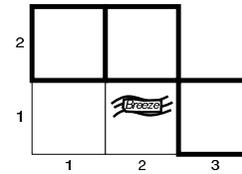
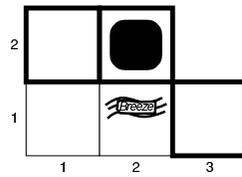
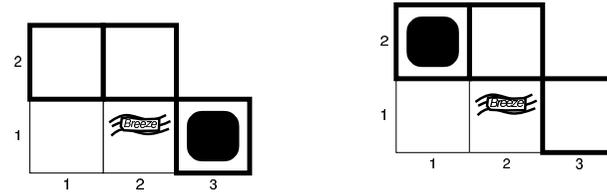
detecting nothing in [1,1],
moving right,
breeze in [2,1]

Consider possible models for “?”s
(considering only pits)

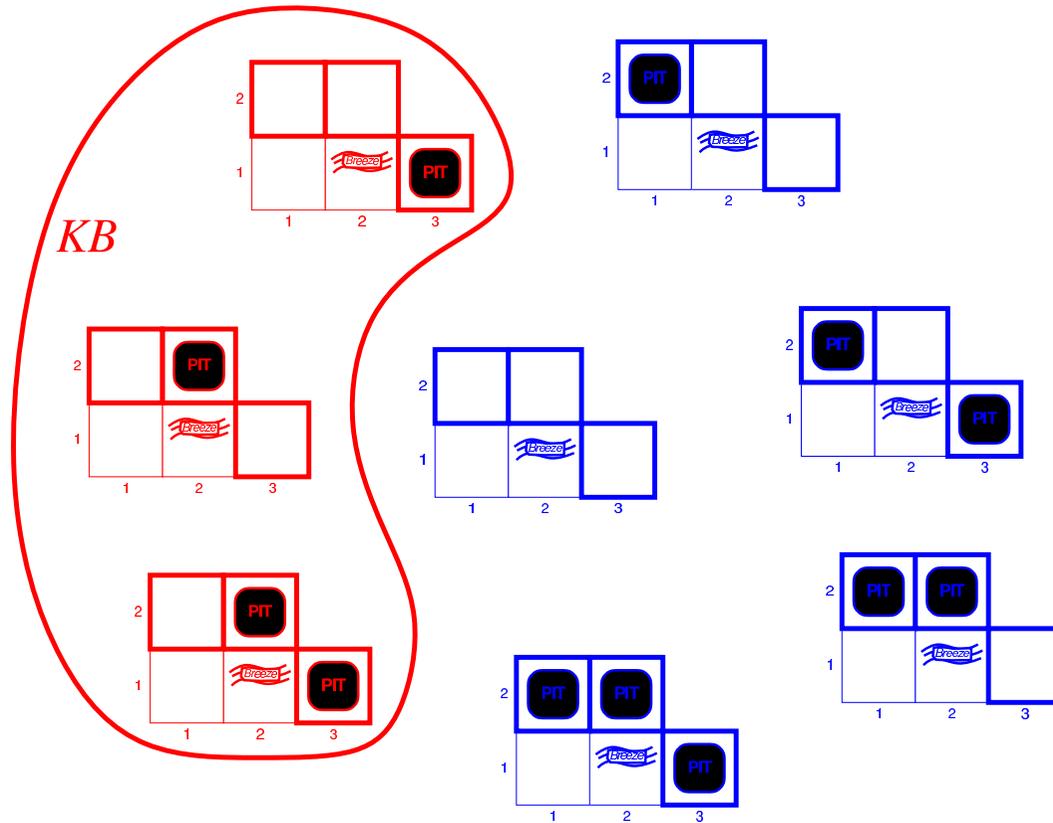
3 Boolean choices
8 possible models



Wumpus Models



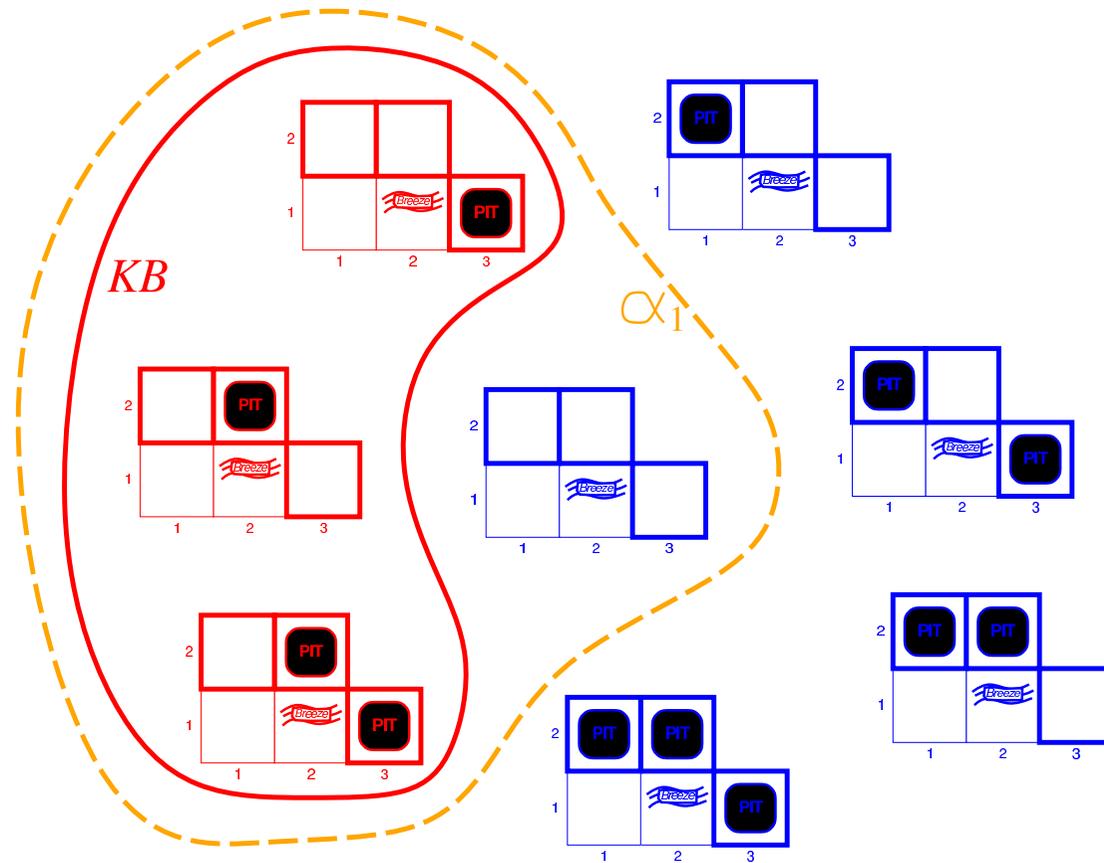
Wumpus Models



KB = wumpus-world rules + observations

Wumpus Models

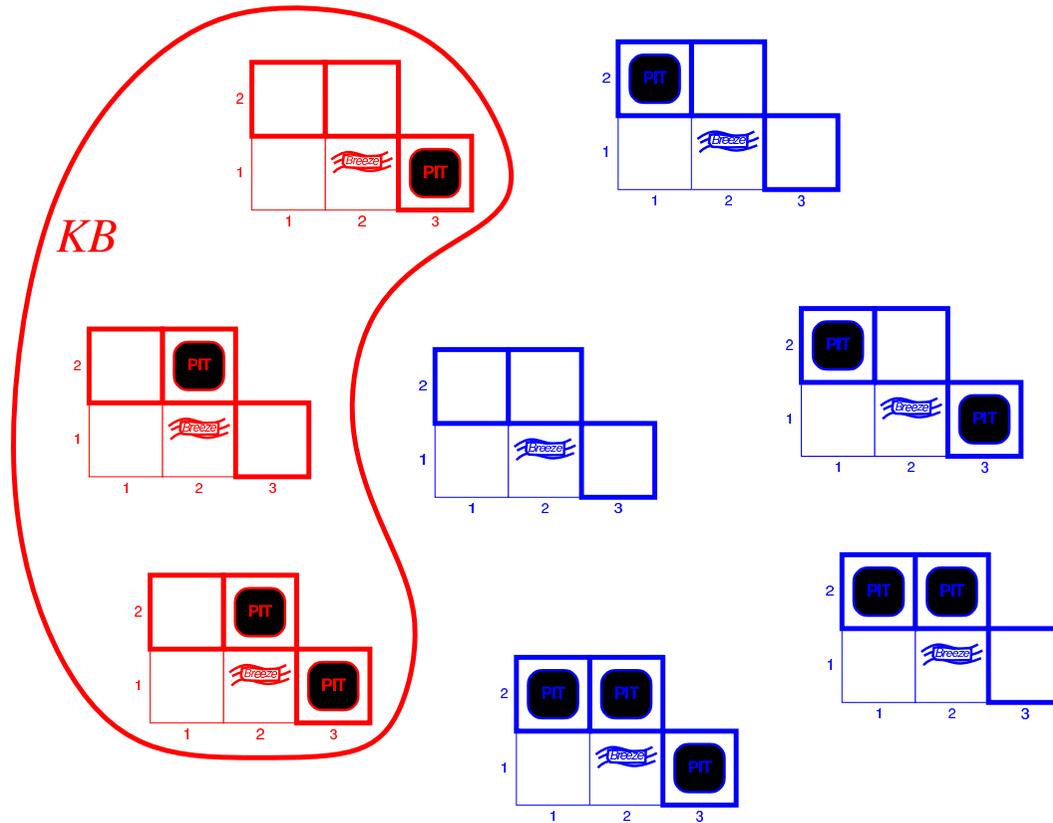
$$KB \models \alpha_1$$



KB = wumpus-world rules + observations

α_1 = “[1,2] is safe”

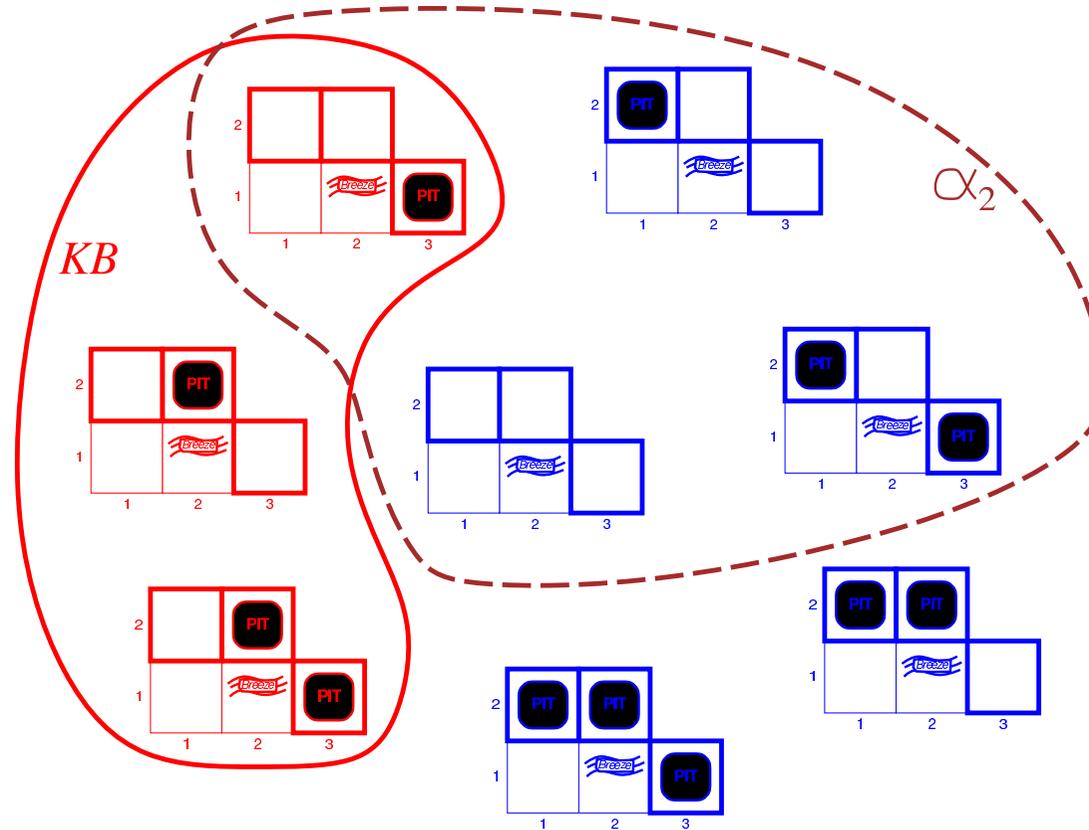
Wumpus Models



KB = wumpus-world rules + observations

Wumpus Models

$$KB \not\models \alpha_2$$



KB = wumpus-world rules + observations

α_2 = “[2,2] is safe”

Inference

Definition

$$KB \vdash_i \alpha$$

means

sentence α can be derived from KB by inference procedure i

Inference

Definition

$$KB \vdash_i \alpha$$

means

sentence α can be derived from KB by inference procedure i

Soundness (of i)

Whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness (of i)

Whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview

First-order Logic

We will define a logic (first-order logic) that

- is expressive enough to say almost anything of interest, and
- for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the *KB*.

Propositional Logic: Syntax

Definition

- Propositional symbols

$A, B, P_1, P_2, \textit{ShirtIsGreen}, \text{ etc.}$

are (atomic) sentences

- If S, S_1, S_2 are sentences, then

$\neg S$ (*negation*)

$S_1 \wedge S_2$ (*conjunction*)

$S_1 \vee S_2$ (*disjunction*)

$S_1 \Rightarrow S_2$ (*implication*)

$S_1 \Leftrightarrow S_2$ (*equivalence*)

are sentences

Propositional logic: Semantics

Propositional Models

Each model specifies true/false for each proposition symbol

Propositional logic: Semantics

Propositional Models

Each model specifies true/false for each proposition symbol

Example

<i>A</i>	<i>B</i>	<i>C</i>
<i>true</i>	<i>true</i>	<i>false</i>

(For three symbols, there are
8 possible models)

Propositional logic: Semantics

Propositional Models

Each model specifies true/false for each proposition symbol

Example

<i>A</i>	<i>B</i>	<i>C</i>
<i>true</i>	<i>true</i>	<i>false</i>

(For three symbols, there are
8 possible models)

Rules for evaluating truth with respect to a model

$\neg S$ is true iff S is false

$S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true

$S_1 \vee S_2$ is true iff S_1 is true or S_2 is true

$S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true

$S_1 \Leftrightarrow S_2$ is true iff S_1 and S_2 have the same truth value

Truth Tables for Connectives

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \Rightarrow B$	$A \Leftrightarrow B$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Wumpus World Sentences

Propositional symbols

$P_{i,j}$ means: “there is a pit in $[i, j]$ ”

$B_{i,j}$ means: “there is a breeze in $[i, j]$ ”

$\neg P_{1,1}$ $\neg B_{1,1}$ $B_{2,1}$

Wumpus World Sentences

Propositional symbols

$P_{i,j}$ means: “there is a pit in $[i, j]$ ”

$B_{i,j}$ means: “there is a breeze in $[i, j]$ ”

$$\neg P_{1,1} \quad \neg B_{1,1} \quad B_{2,1}$$

Sentences

“Pits cause breezes in adjacent squares”

$$P_{1,2} \Rightarrow (B_{1,1} \wedge B_{1,3} \wedge B_{2,2})$$

“A square is breezy **if and only if** there is an adjacent pit”

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

Propositional Inference: Enumeration Method

Example

$$\alpha = A \vee B \quad KB = (A \vee C) \wedge (B \vee \neg C)$$

Checking that $KB \models \alpha$

<i>A</i>	<i>B</i>	<i>C</i>	$A \vee C$	$B \vee \neg C$	<i>KB</i>	α
<i>false</i>	<i>false</i>	<i>false</i>				
<i>false</i>	<i>false</i>	<i>true</i>				
<i>false</i>	<i>true</i>	<i>false</i>				
<i>false</i>	<i>true</i>	<i>true</i>				
<i>true</i>	<i>false</i>	<i>false</i>				
<i>true</i>	<i>false</i>	<i>true</i>				
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<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>			
<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>			
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>			
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>			
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>			
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>			
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>			
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<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>		
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>		
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>		
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>		
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>		
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<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	
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Propositional Inference: Enumeration Method

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<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Propositional Inference: Enumeration Method

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<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Propositional Inference: Enumeration Method

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$$\alpha = A \vee B \quad KB = (A \vee C) \wedge (B \vee \neg C)$$

Checking that $KB \models \alpha$

<i>A</i>	<i>B</i>	<i>C</i>	$A \vee C$	$B \vee \neg C$	<i>KB</i>	α
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Note

Table has 2^n rows for n symbols

Logical Equivalence

Definition

Two sentences are **logically equivalent**, denoted by

$$\alpha \equiv \beta$$

iff they are true in the same models, i.e., iff:

$$\alpha \models \beta \quad \text{and} \quad \beta \models \alpha$$

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Example

$$(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A) \quad \text{(contraposition)}$$

Logical Equivalence

Theorem

If

- $\alpha \equiv \beta$
- γ is the result of replacing a subformula α of δ by β ,

then $\gamma \equiv \delta$

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Example

$$A \vee B \equiv B \vee A$$

implies

$$(C \wedge (A \vee B)) \Rightarrow D \equiv (C \wedge (B \vee A)) \Rightarrow D$$

Important Equivalences

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of \wedge
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of \vee
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of \wedge
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of \vee
$(\alpha \wedge \alpha) \equiv \alpha$	idempotence for \wedge
$(\alpha \vee \alpha) \equiv \alpha$	idempotence for \vee
$\neg\neg\alpha \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	equivalence elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$	de Morgan's rules
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$	de Morgan's rules
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of \wedge over \vee
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of \vee over \wedge

The Logical Constants *true* and *false*

Semantics

true evaluates to *true* in all models

false evaluates to *false* in all models

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true evaluates to *true* in all models

false evaluates to *false* in all models

Important equivalences with *true* and *false*

$$\begin{aligned}(\alpha \wedge \neg\alpha) &\equiv \textit{false} \\(\alpha \vee \neg\alpha) &\equiv \textit{true} && \text{tertium non datur} \\(\alpha \wedge \textit{true}) &\equiv \alpha \\(\alpha \wedge \textit{false}) &\equiv \textit{false} \\(\alpha \vee \textit{true}) &\equiv \textit{true} \\(\alpha \vee \textit{false}) &\equiv \alpha\end{aligned}$$

Validity

Definition

A sentence is **valid** if it is true in **all** models

Examples

$$A \vee \neg A, \quad A \Rightarrow A, \quad (A \wedge (A \Rightarrow B)) \Rightarrow B$$

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Deduction Theorem (connects inference and validity)

$$KB \models \alpha \quad \text{if and only if} \quad KB \Rightarrow \alpha \quad \text{is valid}$$

Satisfiability

Definition

A sentence is **satisfiable** if it is true in **some** model

Examples

$$A \vee B, \quad A, \quad A \wedge (A \Rightarrow B)$$

Satisfiability

Definition

A sentence is **satisfiable** if it is true in **some** model

Examples

$$A \vee B, \quad A, \quad A \wedge (A \Rightarrow B)$$

Definition

A sentence is **unsatisfiable** if it is true in **no** models,
i.e., if it is not satisfiable

Example

$$A \wedge \neg A$$

Satisfiability

Theorem (connects validity and unsatisfiability)

α is valid if and only if $\neg\alpha$ is unsatisfiable

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Theorem (connects inference and unsatisfiability)

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Satisfiability

Theorem (connects validity and unsatisfiability)

α is valid if and only if $\neg\alpha$ is unsatisfiable

Theorem (connects inference and unsatisfiability)

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

Note

Validity and inference can be proved by **reductio ad absurdum**

Two Kinds of Proof Methods

1. Application of inference rules

Legitimate (sound) generation of new sentences from old

Construction of / search for a proof

(proof = sequence of inference rule applications)

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Different kinds

Tableau calculus, resolution, forward/backward chaining, ...

Two Kinds of Proof Methods

2. Model checking

Construction of / search for a satisfying model

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2. Model checking

Construction of / search for a satisfying model

Different kinds

- Truth table enumeration (always exponential number of symbols)
- Improved backtracking search for models
e.g.: Davis-Putnam-Logemann-Loveland
- Heuristic search in model space (sound but incomplete)
e.g.: hill-climbing algorithms