
Program analysis in KeY

Tobias Gedell

`gedell@cs.chalmers.se`

Outline

- ▷ Who am I?
- ▷ Some words on static program analysis
- ▷ Reachable definitions analysis
- ▷ Why is this interesting for KeY?
- ▷ Presentation of mini-project
- ▷ Demo of mini-project

Work in progress!

This is work in progress!

Who am I?

My position:

- ▷ First year PhD student at Chalmers
- ▷ Supervised by Reiner Hähnle

My main interests are:

- ▷ Program languages
- ▷ Program analysis

What I have done in the past:

- ▷ Written a compiler for the functional language STG.
- ▷ Worked on and implemented a cache sensitive Haskell compiler.
- ▷ Worked on and implemented a usage analysis for Haskell in GHC.

Static program analysis

Program analyses are designed to calculate properties about programs. For example:

- ▷ Which variables are live at a certain program point?
- ▷ What values can a variable hold at a certain program point?
- ▷ Which functions are going to be called during program execution?

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Program analysis

cheap

low precision

Verification (TP)

expensive

high precision

Reaching definitions

An assignment is reachable at a certain program point if there exist a program trace such that the variable has not been reassigned when the point is reached.

Program: `a = 1; b = 1; if(cond) {b = 1; } else {b = 1; }`

Reaching definitions

An assignment is reachable at a certain program point if there exist a program trace such that the variable has not been reassigned when the point is reached.

Program: $\underbrace{a = 1;}_0 \underbrace{b = 1;}_1 \underbrace{\text{if}(\text{cond}) \overbrace{\{b = 1;\}}^3 \text{ else } \overbrace{\{b = 1;\}}^4}_2$

Reaching definitions

An assignment is reachable at a certain program point if there exist a program trace such that the variable has not been reassigned when the point is reached.

Program: $a = 1;$ $b = 1;$ $\text{if}(\text{cond}) \{b = 1;\}$ $\text{else} \{b = 1;\}$

The diagram shows the program code with brackets underneath indicating reaching definitions. Under 'a = 1;' is a bracket labeled '0'. Under 'b = 1;' is a bracket labeled '1'. Under the entire 'if-else' block is a bracket labeled '2'. Above the 'if' block is a bracket labeled '3'. Above the 'else' block is a bracket labeled '4'.

A solution: $\{(a, 0), (b, 3), (b, 4)\}$

Reaching definitions

An assignment is reachable at a certain program point if there exist a program trace such that the variable has not been reassigned when the point is reached.

Program: $a = 1;$ $b = 1;$ $\text{if}(\text{cond}) \{b = 1;\}$ $\text{else} \{b = 1;\}$

The diagram shows the program code with brackets underneath indicating reaching definitions. A bracket labeled '0' is under 'a = 1;'. A bracket labeled '1' is under 'b = 1;'. A bracket labeled '2' spans the entire conditional block. Inside the conditional block, a bracket labeled '3' is under '{b = 1;}' in the 'if' branch, and a bracket labeled '4' is under '{b = 1;}' in the 'else' branch.

A solution: $\{(a, 0), (b, 3), (b, 4)\}$

The solution must be a [safe approximation](#)! But we want to get the least solution possible.

Why is this interesting for KeY?

Program analysis can be used by the KeY system in at least two ways:

- ▷ We can integrate analyses to for example reduce branching.

```
void m1(SomeClass o)
{
    ...
    o.m2( );
    ...
}
```

Why is this interesting for KeY?

Program analysis can be used by the KeY system in at least two ways:

- ▷ We can integrate analyses to for example reduce branching.

```
void m1(SomeClass o)
{
    ...
    o.m2( );
    ...
}
```

- ▷ We can try to build a framework based on abstract interpretation which could allow the user to abstract away from certain things when making the proof. (This idea is still a little vague, more theoretical work is needed.)

Mini-project

In order to get going I started with trying to implement the reaching definitions analysis in KeY.

I choose to implement it for the WHILE language.

program ::= *stmt*

stmt ::= *var*^{*loc*} = *term*;
| **if**^{*loc*}(*term*) *stmt* **else** *stmt*
| **while**^{*loc*}(*term*) *stmt*
| {*stmt**}

term ::= *literal*
| *var*
| *term* *op* *term*
| (*term*)

Reaching definitions - rules

$S \subseteq \text{Var} \times \text{Loc}$

```
functions {  
  VarSet Union(VarSet, VarSet);  
  VarSet Singleton(Quoted, ...);  
  VarSet CutVar(VarSet, Quoted);  
  VarSet Empty;  
}
```

Reaching definitions - rules

RULE

$$\frac{\dots}{S_0 \vdash stmt \Downarrow S_1}$$

```
predicates {  
    wrapper(VarSet, VarSet);  
}
```

```
rules {  
    rdef_rule {  
        find (==> diamond{{stmt}})(wrapper(vs0, vs1))  
        ...  
    };  
}
```


Reaching definitions - rules

ASSIGN

$$S \vdash x \stackrel{loc}{=} v; \Downarrow \underbrace{(S \setminus \{x, l \mid l \in Loc\}) \cup \{x, loc\}}_{\substack{\text{Remove previous} \\ \text{assignments of } x}}$$

```
rdef_assign {
  find (==> diamond{{#var = #se;}}
        (wrapper(vs, Union( CutVar(vs, #var),
                             Singleton(#var, ...))))))
  close goal
};
```

Reaching definitions - rules

IF

$$\frac{S_0 \vdash stmt_0 \Downarrow S_1 \quad S_0 \vdash stmt_1 \Downarrow S_2}{S_0 \vdash \mathbf{if} (e) stmt_0 \mathbf{else} stmt_1 \Downarrow S_1 \cup S_2}$$

← Approximation!

```
rdef_if_else {
  find (==> diamond{{if(#se) #stmt0 else #stmt1}}
        (wrapper(vs0, Union(vs1, vs2))))
  replacewith (==> diamond{{#stmt0}}(wrapper(vs0, vs1)));
  replacewith (==> diamond{{#stmt1}}(wrapper(vs0, vs2)));
};
```

Getting the variable names

First problem:

- ▷ I need to store variable names in formulas, which is not possible since variable names are flexible terms.

Solved by adding a quoting mechanism. With it you can say:
`varcond (#qvar quotes #var)`, which adds a constraint.

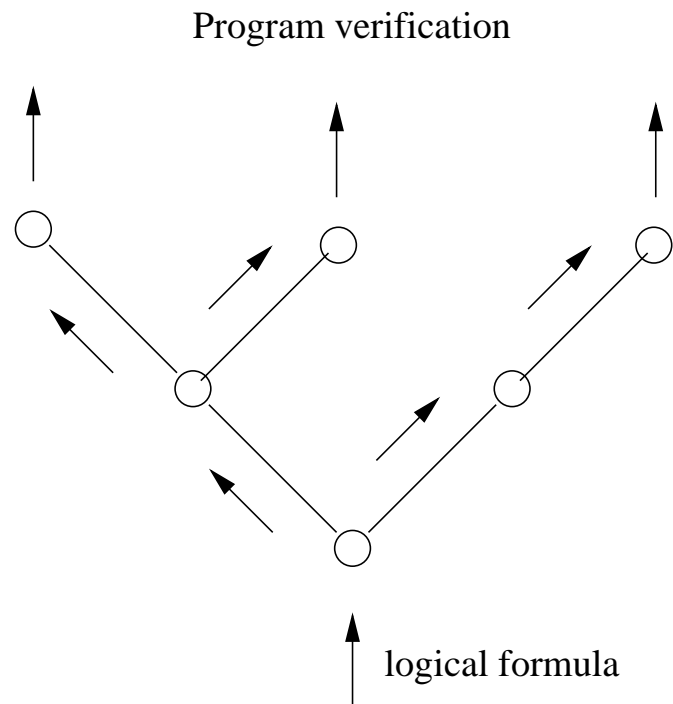
```
schema variables { Quoted #qvar; }

rdef_assign {
  find (==> diamond{{#var = #se;}}
        (wrapper(vs, Union( CutVar(vs, #qvar),
                              Singleton(#qvar, ...))))))
  varcond (#qvar quotes #var)
  close goal
};
```

Flow of information

Second problem:

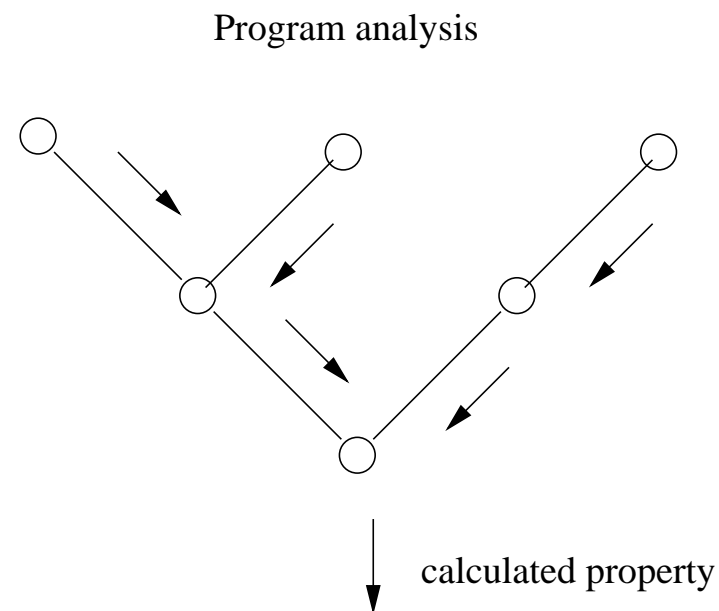
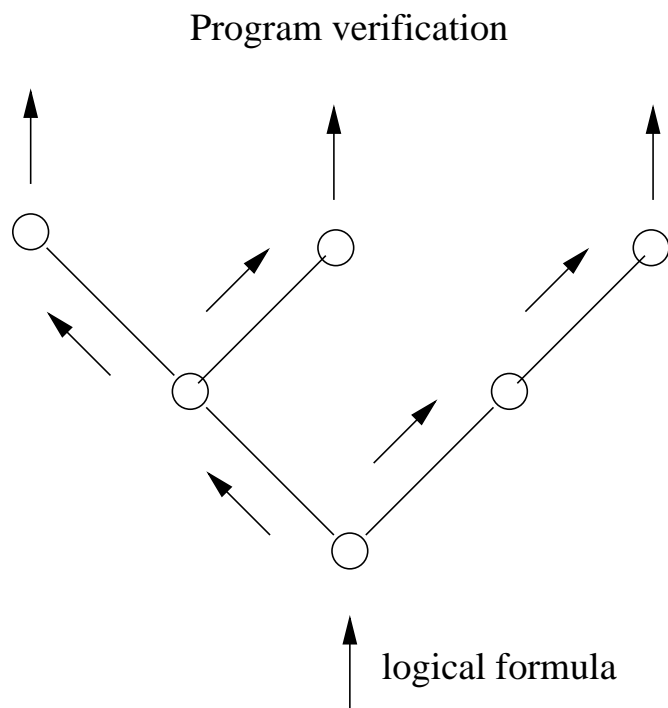
- ▷ Flow of information.



Flow of information

Second problem:

- ▷ Flow of information.

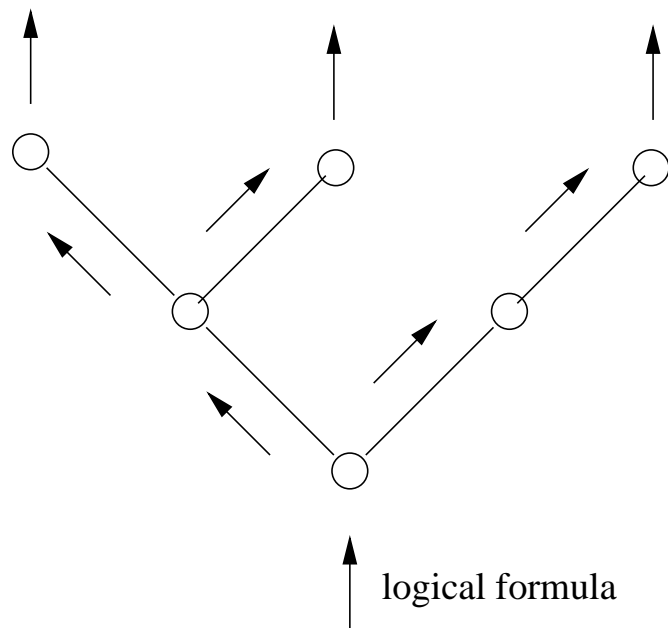


Flow of information

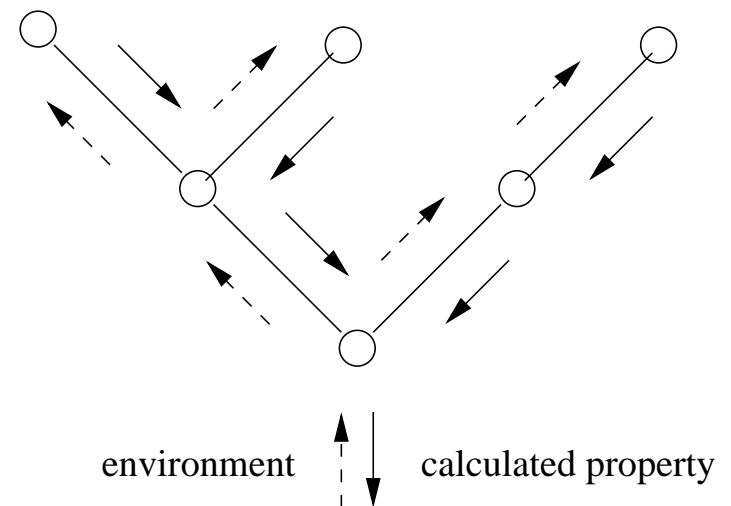
Second problem:

- ▷ Flow of information.

Program verification



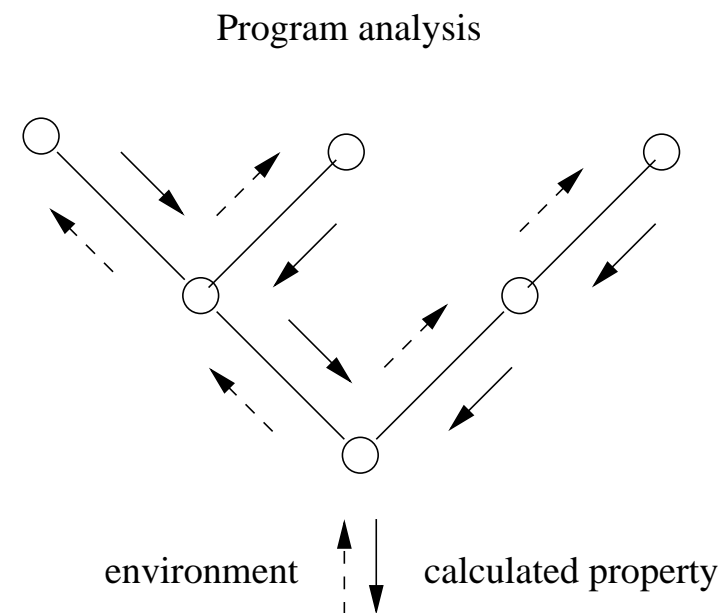
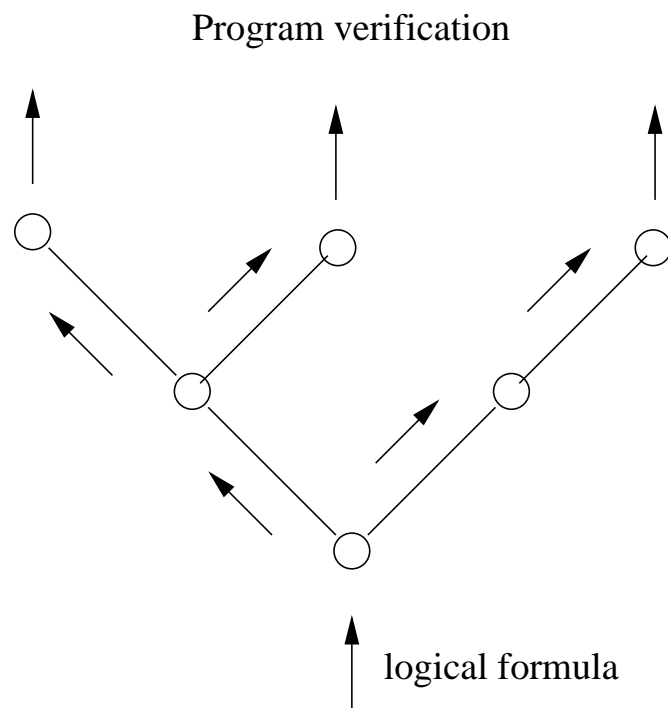
Program analysis



Flow of information

Second problem:

- ▷ Flow of information.



Program verification is like type checking while reaching definitions analysis is more like type inferens.

Flow of information

Meta variables solves the information flow problem.

```
problem {  
  <{ if(cond) { a = 1; } else { b = 1; } }>  
  wrapper(Empty, Union(..., ...))  
}
```

Now I must give the solution...

Flow of information

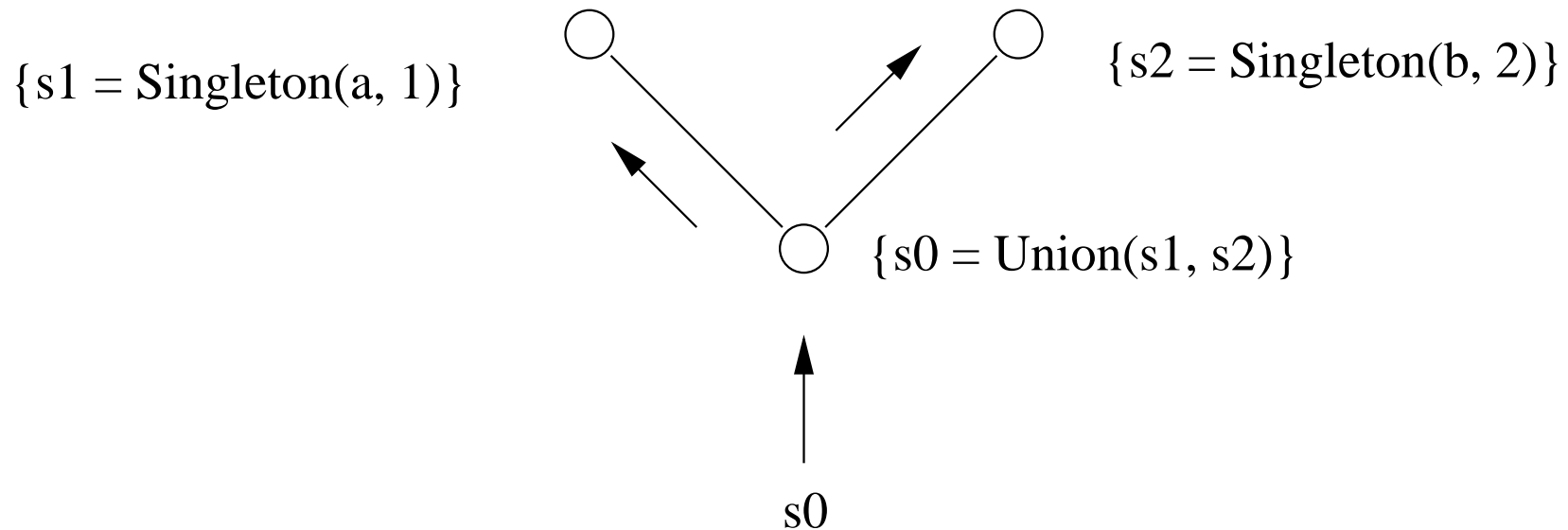
Meta variables solves the information flow problem.

```
problem {  
  ex s:VarSet.  
    <{ if(cond) { a = 1; } else { b = 1; } }>  
    wrapper(Empty, s)  
}
```

Now the system checks if there exists a solution!

Flow of information

```
problem {  
  ex s:VarSet.  
  <{ if(cond) { a = 1; } else { b = 1; } }>  
  wrapper(..., s)  
}
```



If I just get hold of the collection of constraints I can find the solution!

Demo

Time for a demo!

Demo - example 1

Program:

$$\underbrace{a = 1;}_0 \underbrace{b = 1;}_1 \underbrace{\text{if}(\text{cond}) \overbrace{\{b = 1;\}}^3}_2$$

Solution:

$\{(a, 0), (b, 1), (b, 3)\}$

Demo - example 2

Program:

$$\underbrace{a = 1;}_0 \underbrace{b = 1;}_1 \underbrace{\text{if}(\text{cond}) \overbrace{\{b = 1;\}}^3 \text{ else } \overbrace{\{b = 1;\}}^4 \overbrace{\{c = 1;\}}^5}_{2}$$

Solution:

$\{(a, 0), (b, 3), (b, 4), (c, 5)\}$

Demo - example 3

Program:

$$\underbrace{a = 1;}_0 \underbrace{b = 1;}_1 \underbrace{\text{if}(\text{cond}) \{ \underbrace{b = 1;}_3 \} \text{ else } \{ \underbrace{b = 1;}_4 \underbrace{c = 1;}_5 \} \text{ while}(\text{cond}) \underbrace{c = 1;}_7 \}_2}_6$$

Solution:

Entry_0 = {}
Entry_1 = {(a, 0)}
Entry_2 = {(a, 0), (b, 1)}
Entry_3 = {(a, 0), (b, 1)}
Entry_4 = {(a, 0), (b, 1)}
Entry_5 = {(a, 0), (b, 4)}
Entry_6 = {(a, 0), (b, 4), (c, 5), (c, 7)}
Entry_7 = {(a, 0), (b, 4), (c, 5), (c, 7)}
Exit_0 = {(a, 0)}
Exit_1 = {(a, 0), (b, 1)}
Exit_2 = {(a, 0), (b, 3), (b, 4), (c, 7)}
Exit_3 = {(a, 0), (b, 3)}

...

Conclusion

Conclusion:

- ▷ It is possible to implement syntax directed program analyses in KeY. (At least for simple languages.)
- ▷ It would be nice to do computation in the taclets.
- ▷ To me KeY + meta variables + constraints seems very similar to for example Prolog.

Future work

I will now look into:

- ▷ What analyses are useful to KeY?
- ▷ Can we create a framework based on abstract interpretation?
- ▷ If we can, how can the framework be used?