# Interactive Verification of Concurrent Systems

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## Overview

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- Analysis (mostly) through model checking (e.g. the FDR model checker)

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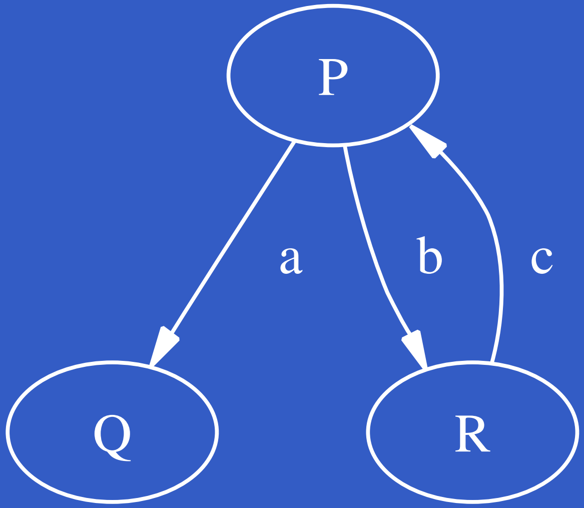
- Communication is a sequence of atomic *events* (e.g. Processes $\triangleq$ Languages)

## Automata as Processes

- States of automata can be regarded as processes:



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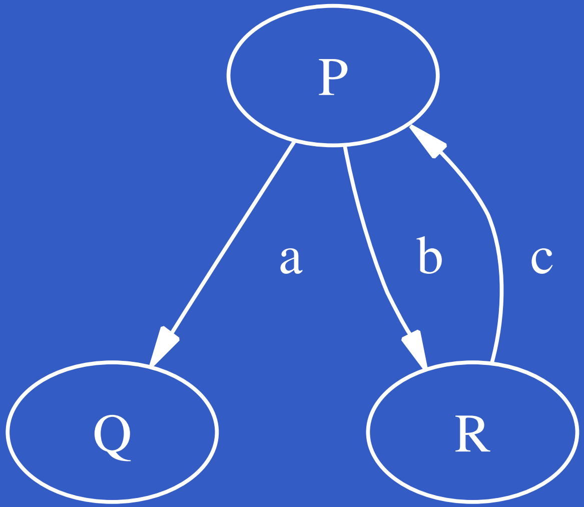

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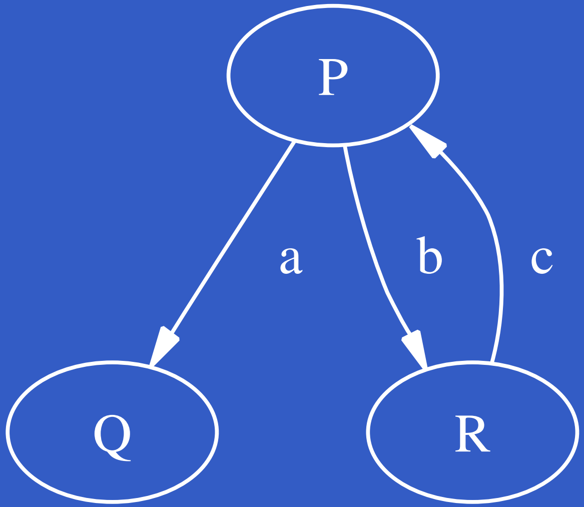

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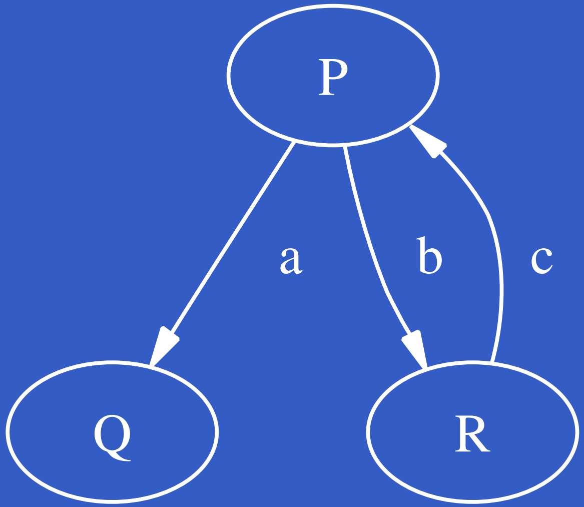

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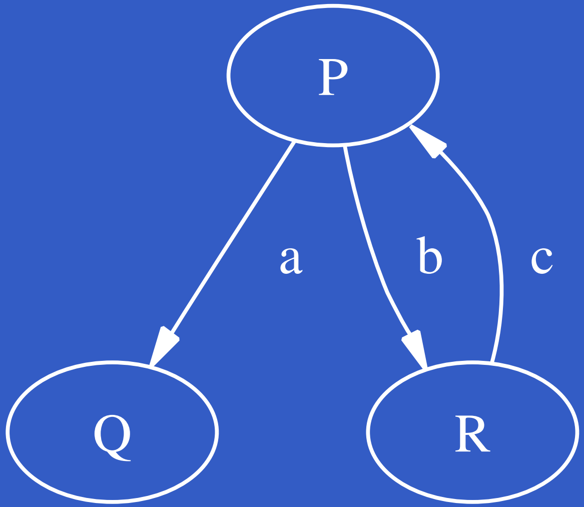

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- Generalised as *labelled transition systems* (LTS), usually infinite

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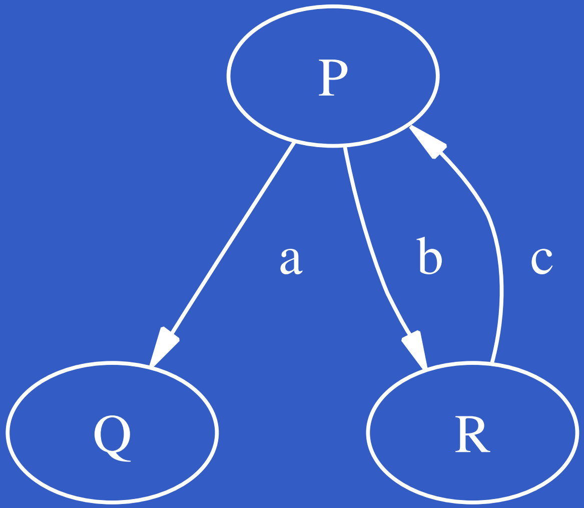

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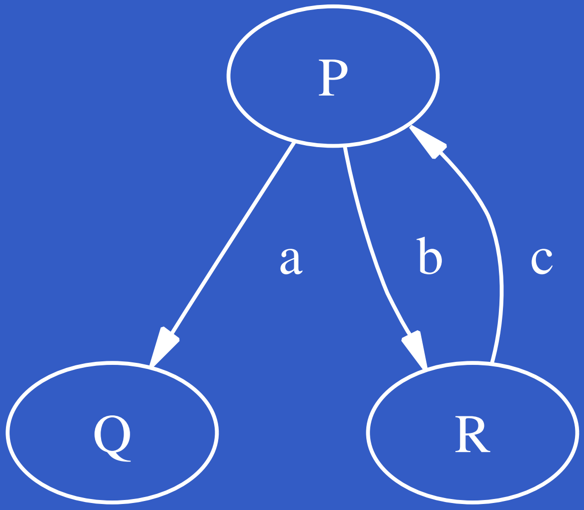

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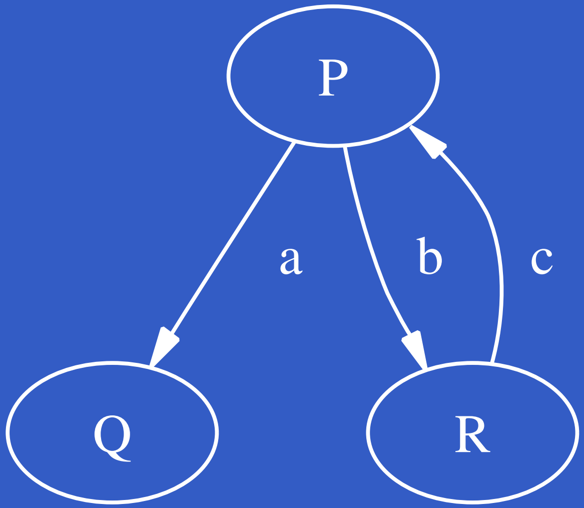

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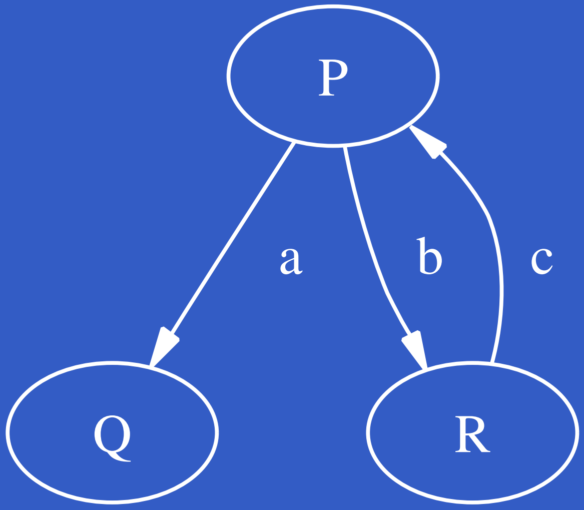

$$\begin{aligned} Q &= \mathit{stop} \\ R &= c \longrightarrow P \\ P &= (a \longrightarrow Q) \,\Box\, (b \longrightarrow R) \end{aligned}$$

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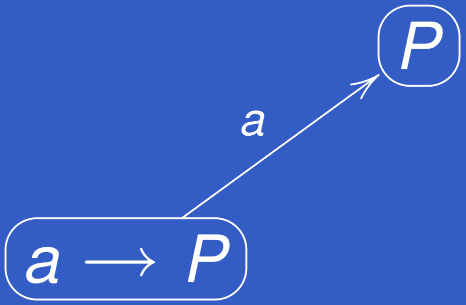

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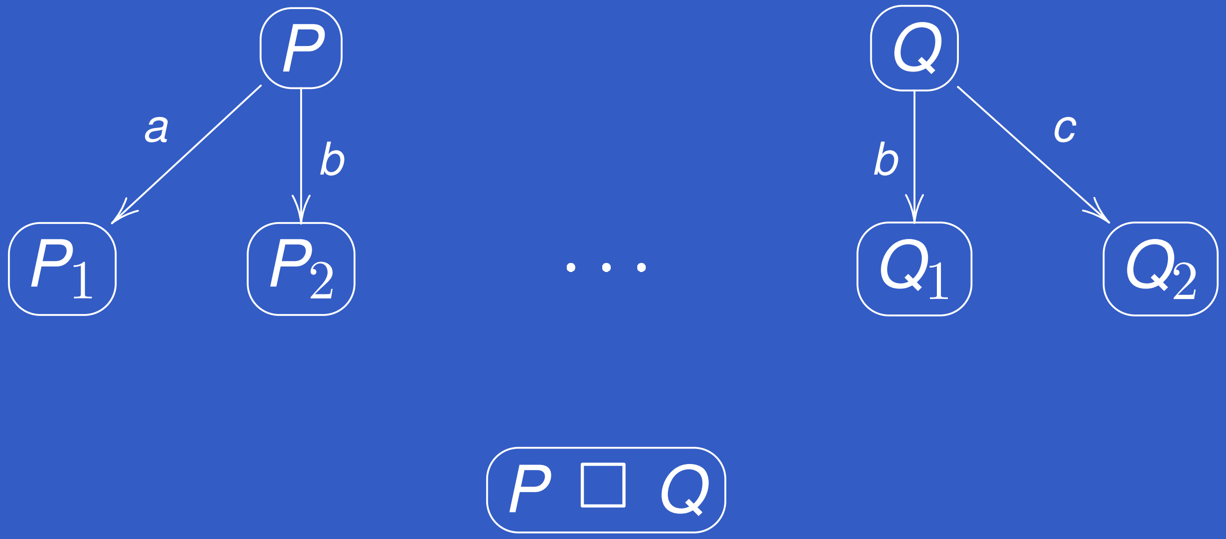

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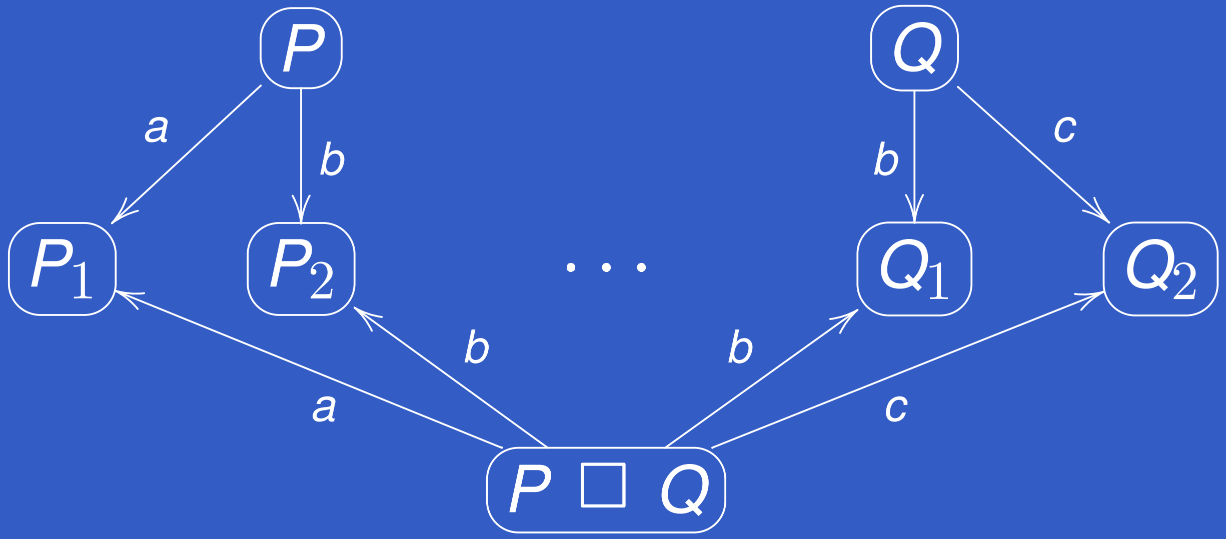
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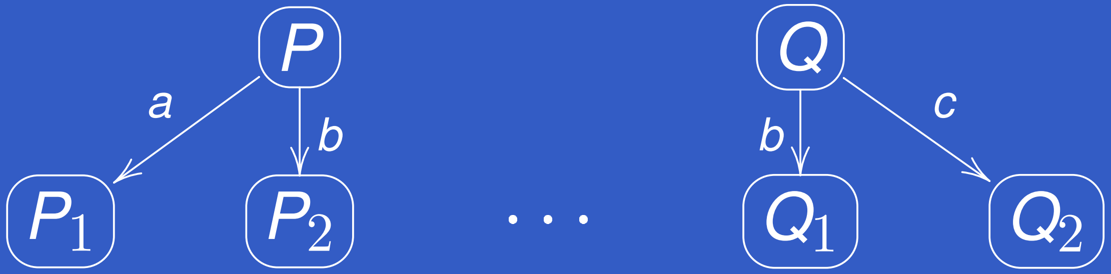
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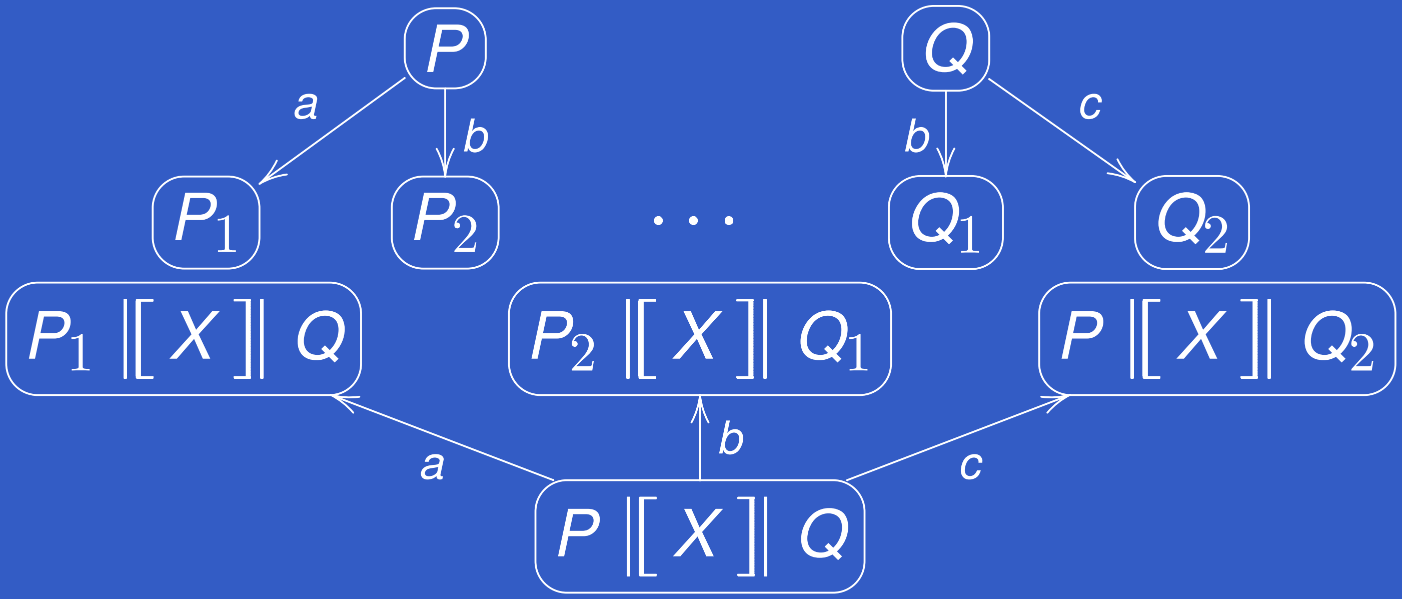

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- Parallelism $\Longrightarrow$ Product of LTSs
- Full synchronisation $P \,[\![\, A I \,]\!]\, Q \Longrightarrow$ Intersection of languages
- Shorter notation for interleaving:

$$P \,|\!|\!|\, Q \quad := \quad P \,[\![\, \emptyset \,]\!]\, Q$$

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- Reading of messages: Generalised Choice

$$?x : A \longrightarrow P(x) :=$$
$$(v_1 \longrightarrow P(v_1)) \; \Box \; (v_2 \longrightarrow P(v_2)) \; \Box \; \cdots$$

(where $A = \{v_1, v_2, \ldots\}$)

## Example

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- LTS view:

$$\begin{tikzcd}
 & S \arrow[dl, bend right, "0"'] \arrow[dr, bend right, "n"'] & & \\
S_0 \arrow[ur, bend right, "1"'] & \cdots & S_n \arrow[ul, bend right, "n+1"'] & \cdots
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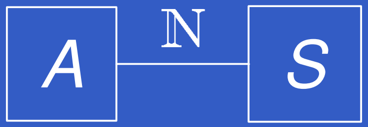

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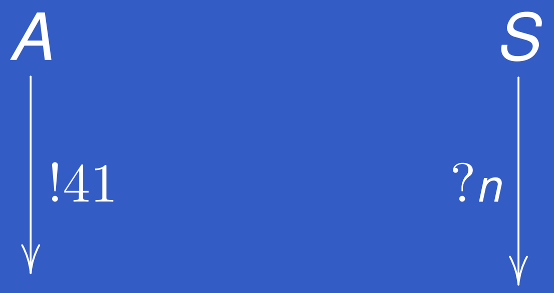

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$$\begin{tikzcd}
A \arrow[d, dash] & & S \arrow[d, dash] \\
41 \arrow[rr, dotted] \arrow[d] & & 41 \arrow[d] \\
?n \longrightarrow A(n) \arrow[d, "?n"] & & !42 \longrightarrow S \arrow[d, "!42"'] \\
{} & & {}
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A(42) \arrow[d, dotted] & & & S \arrow[d, dotted] \\
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## CSP for Java (JCSP)

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- Very similar to the Occam language

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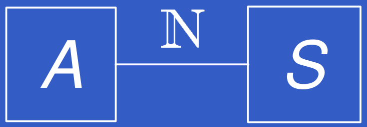

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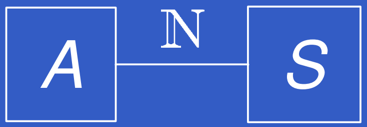
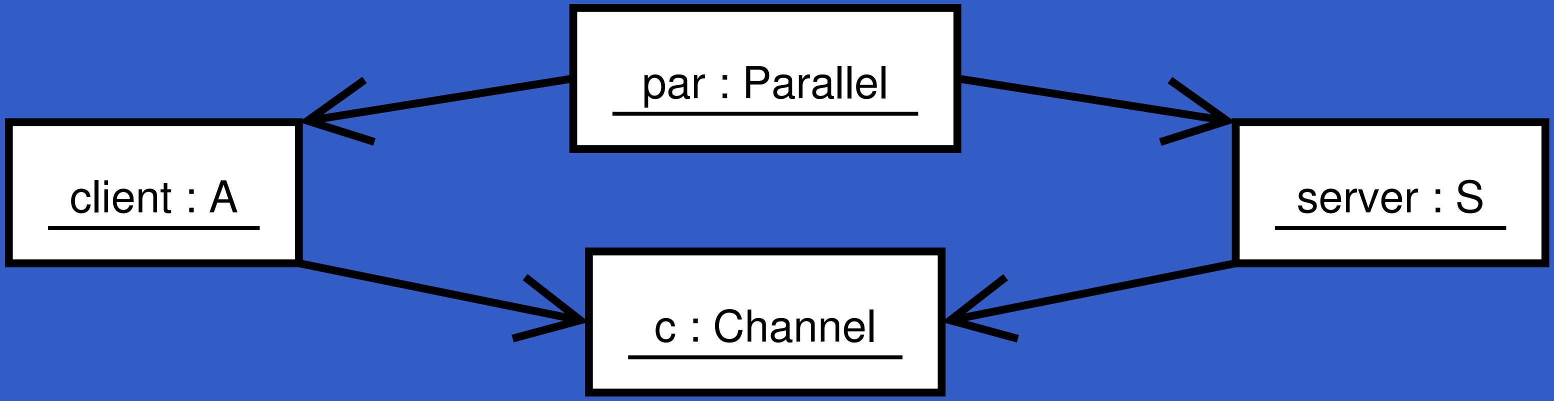
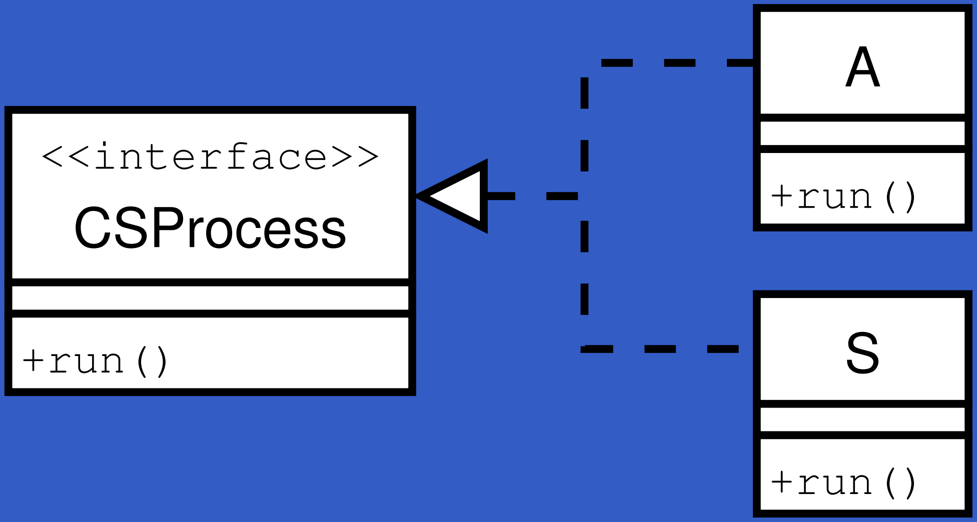


- Corresponding JCSP object diagram:



## Introduction to JCSP

- Processes represented by interface



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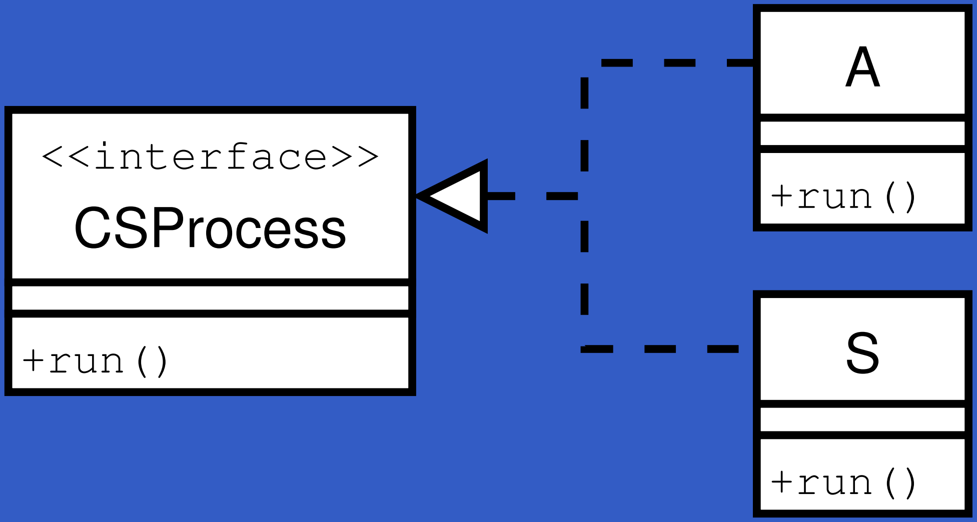
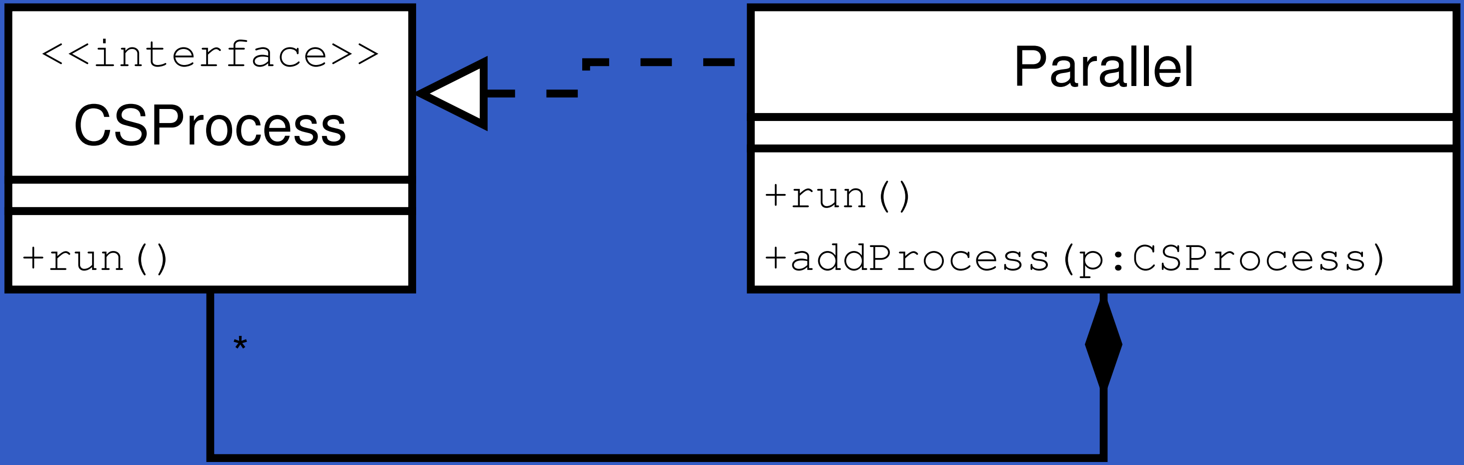


- In JCSP processes have identities

## Introduction to JCSP (2)

- CSP operator of parallelism (actually interleaving) captured by



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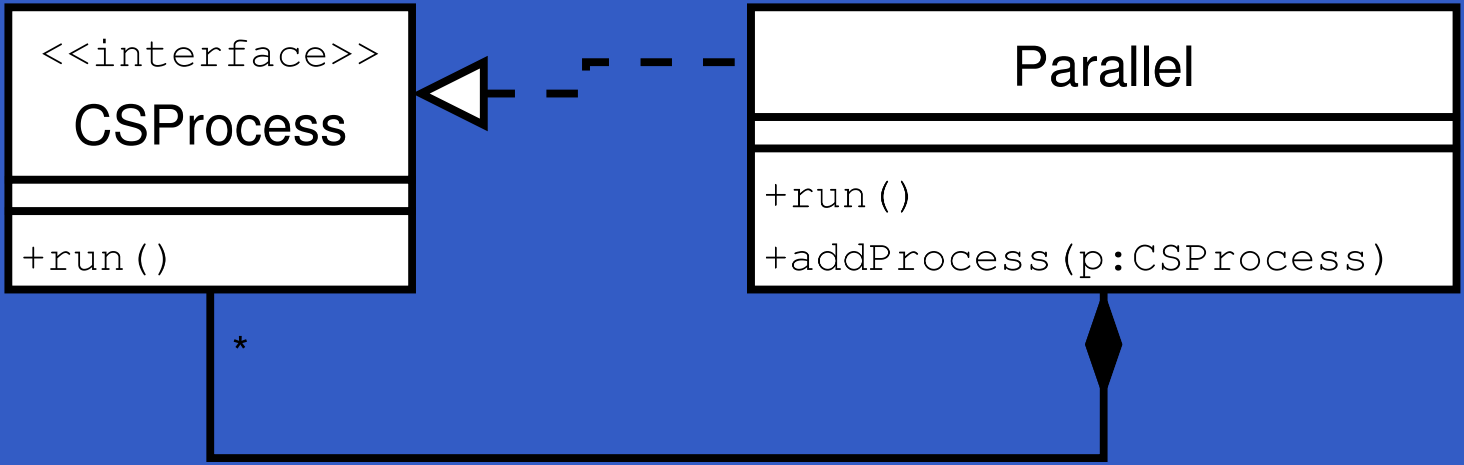
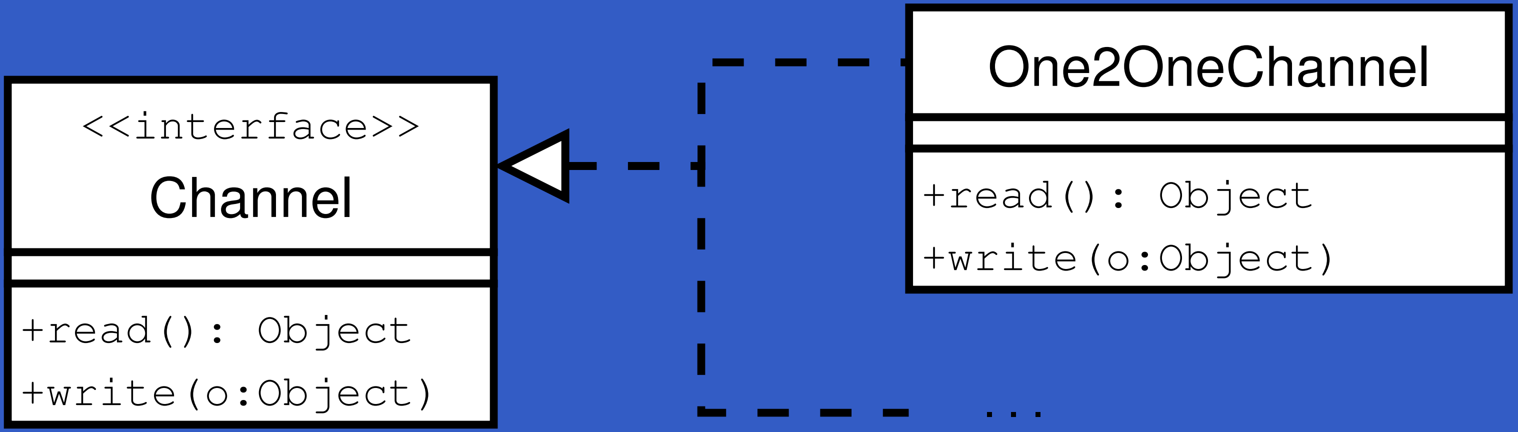


- Each process is executed in its own thread

## Introduction to JCSP (3)

- Messages are sent through *channels*



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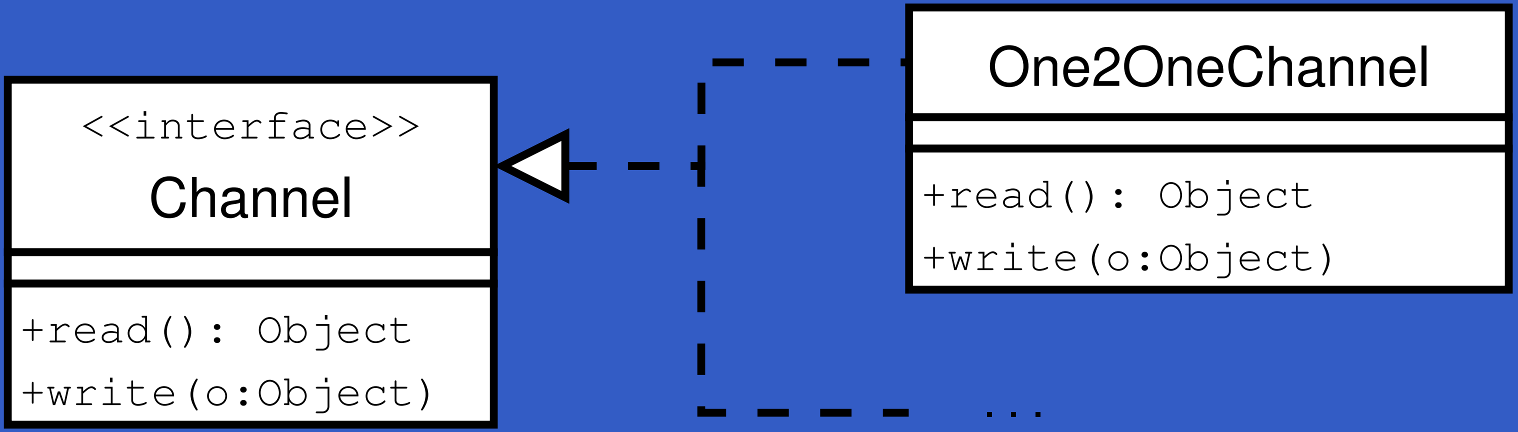

- No “unbound” events as in CSP

## Implementation of $S$ in JCSP

```
import jcsp.lang.*;
public class S implements CSProcess {
    private final Channel c;
    public S (Channel c) { this.c = c; }

    public void run () {
        while ( true ) {
            final Integer i = (Integer)c.read();
            c.write(new Integer(i.intValue() + 1));
        }
    }
}
```

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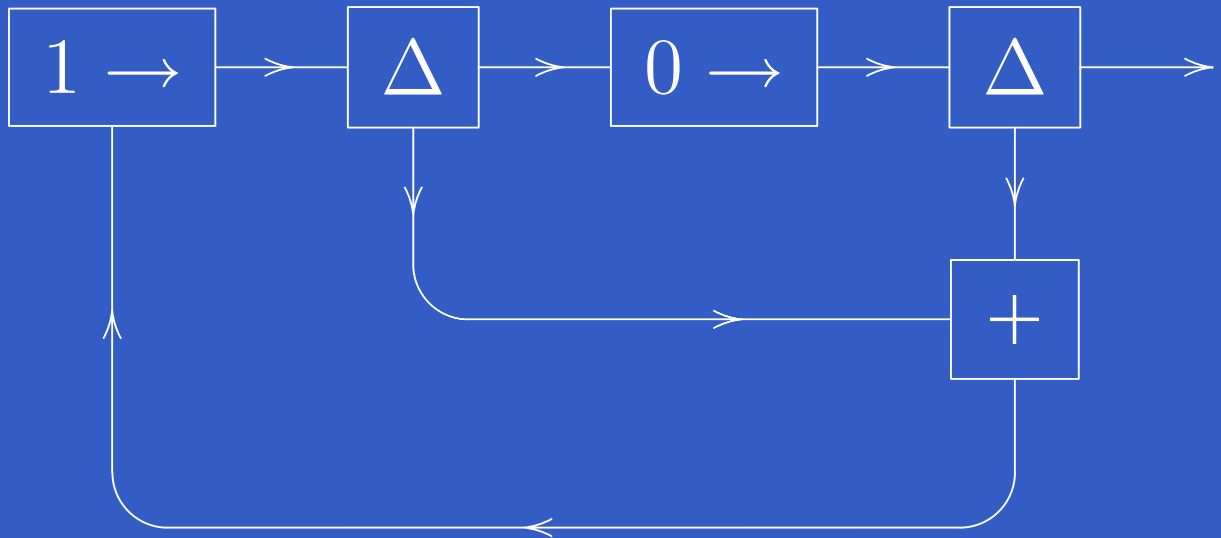

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| | |
|---|---|
| JavaCard(DL) | CSP model of JCSP |
| CSP calculus | |
| Modal logic/calculus | |

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  - $\Longrightarrow$ no shared memory
- JCSP primitives are modelled using CSP operators

## Modelling JCSP Parallelism

- Class Parallel is represented by interleaving:

```
Parallel par = new Parallel();
par.addProcess(s1);
par.addProcess(s2); ...
par.run();
```

## Modelling JCSP Parallelism

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  par.run();
```

- Corresponding process term:

$$T(\text{.. par.run(); \ldots}) =$$
$$T(\text{.. s1.run(); \ldots}) \;|||\; T(\text{.. s2.run(); \ldots}) \;|||\; \cdots$$

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- Execution of Channel.read(), Channel.write() raises events

## Example

System after execution of

```
Channel c = new One2OneChannel();
Parallel par = new Parallel();
par.addProcess(new P1 (c));
par.addProcess(new P2 (c));
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## Example

System after execution of

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Parallel par = new Parallel();
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is described by

$$\text{O2ORouter} \;[\![\, \text{O2OEvents} \,]\!]\; \big( \mathcal{T}(\text{p1.run();}) \;|||\; \mathcal{T}(\text{p2.run();}) \big)$$

## Message Transmission through $c$

$$\text{O2ORouter} \,|[\, \text{O2OEvents} \,]|\, \big( T(\text{.. c.write(o); \ldots}) \;|||\; T(\text{.. a=c.read(); \ldots}) \big)$$

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$$\rightsquigarrow \quad \cdots [\![ \cdots ]\!] \big( \,!o \longrightarrow T(\text{.. ...}) \;|||\; ?a \longrightarrow T(\text{.. ...}) \big)$$

## Message Transmission through $c$

$$\text{O2ORouter} \;[\![\, \text{O2OEvents} ]\!]\; \Big( T(\text{.. c.write(o); ...}) \;|||\; T(\text{.. a=c.read(); ...}) \Big)$$

$$\rightsquigarrow \quad \cdots [\![ \cdots ]\!] \Big( \,!o \longrightarrow T(\text{.. ...}) \;|||\; ?a \longrightarrow T(\text{.. ...}) \Big)$$

Further execution is performed by CSP calculus:

$$\rightsquigarrow \quad \cdots$$

$$\rightsquigarrow \quad !o \longrightarrow \Big( \text{O2ORouter} \;[\![ \cdots ]\!]\; \big( T(\text{.. ...}) \;|||\; T(\text{.. ...}) \big) \Big)$$

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- Incomplete: More complex communication (e.g. buffered channels, barriers, sending of complex data structures)

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  - Treatment of proving techniques (postponed): Induction, compositional proving