Inferring Invariants by Static Analysis in KeY

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Automatically find invariants:

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- In particular: Loop invariants
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Do so within KeY itself.

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while(i < a.length) {
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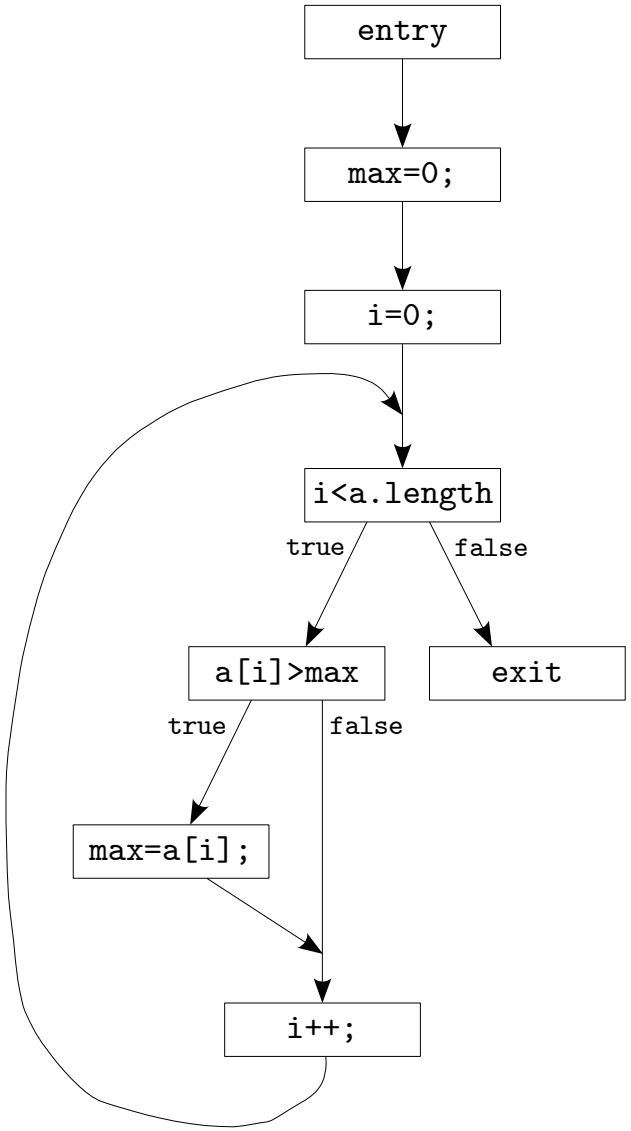

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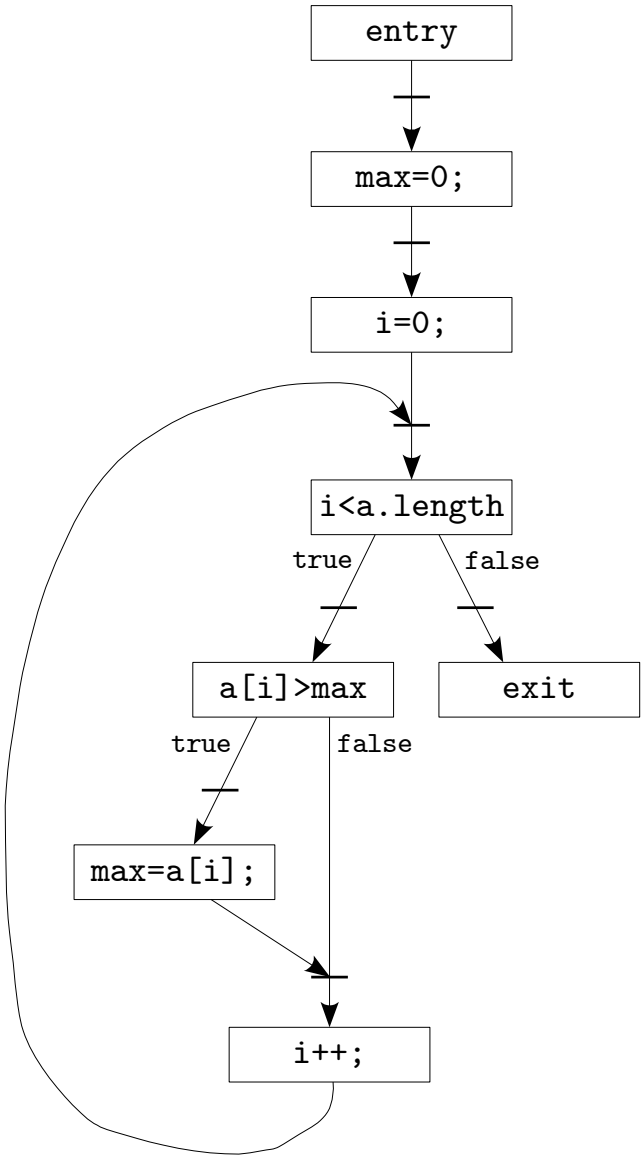

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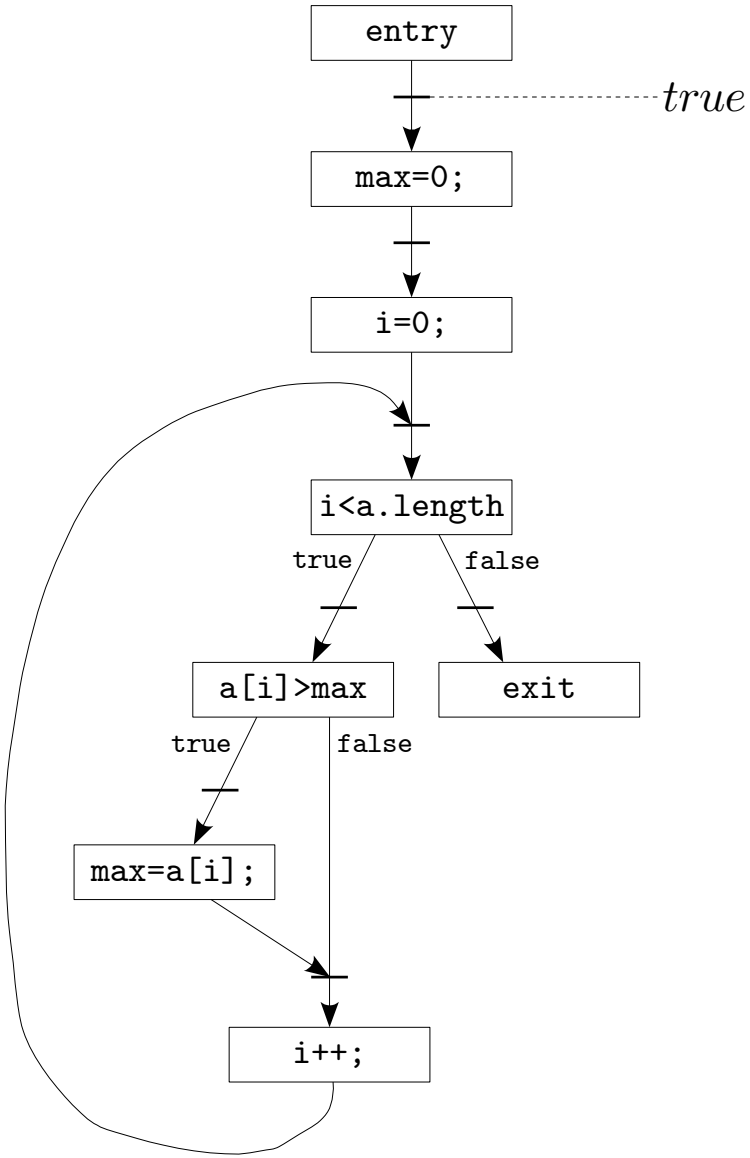

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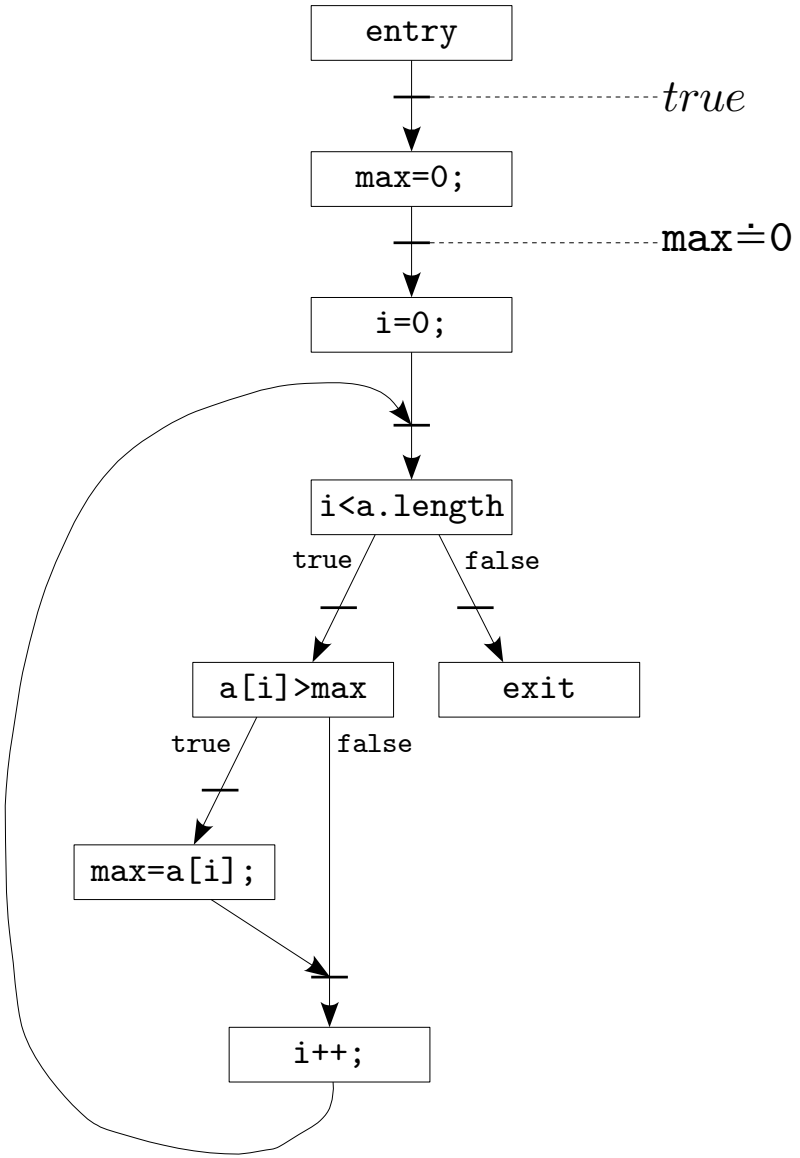

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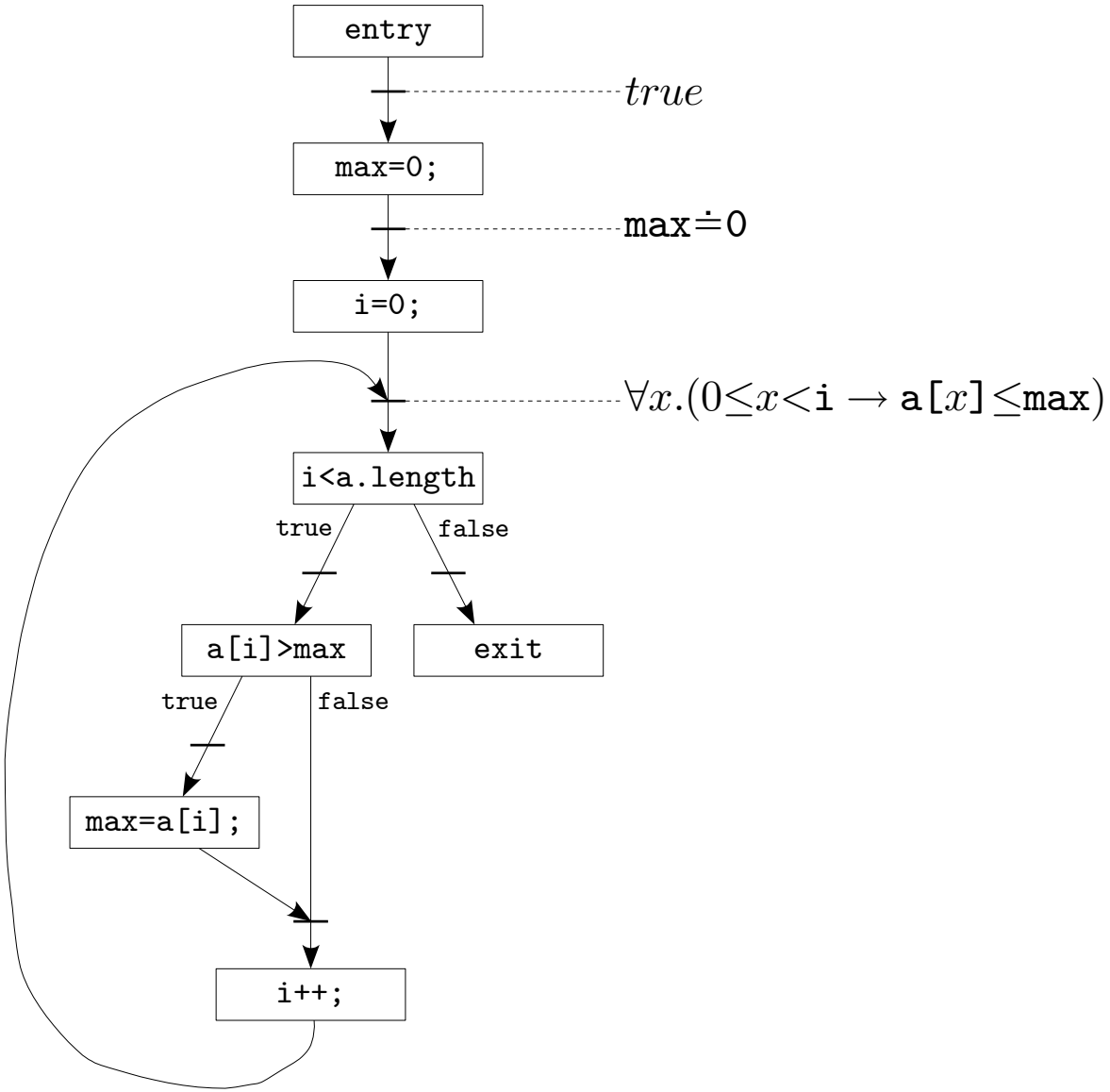

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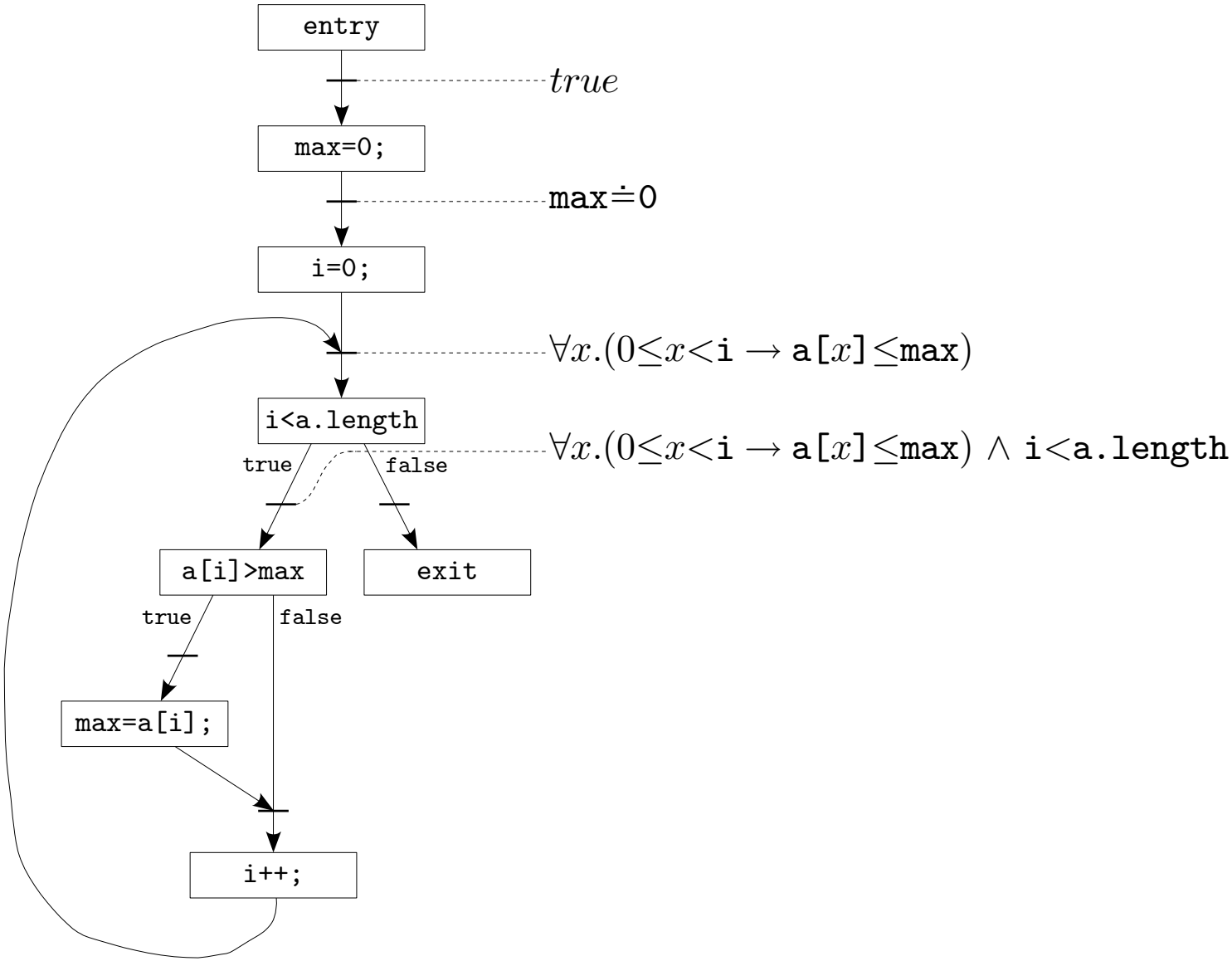

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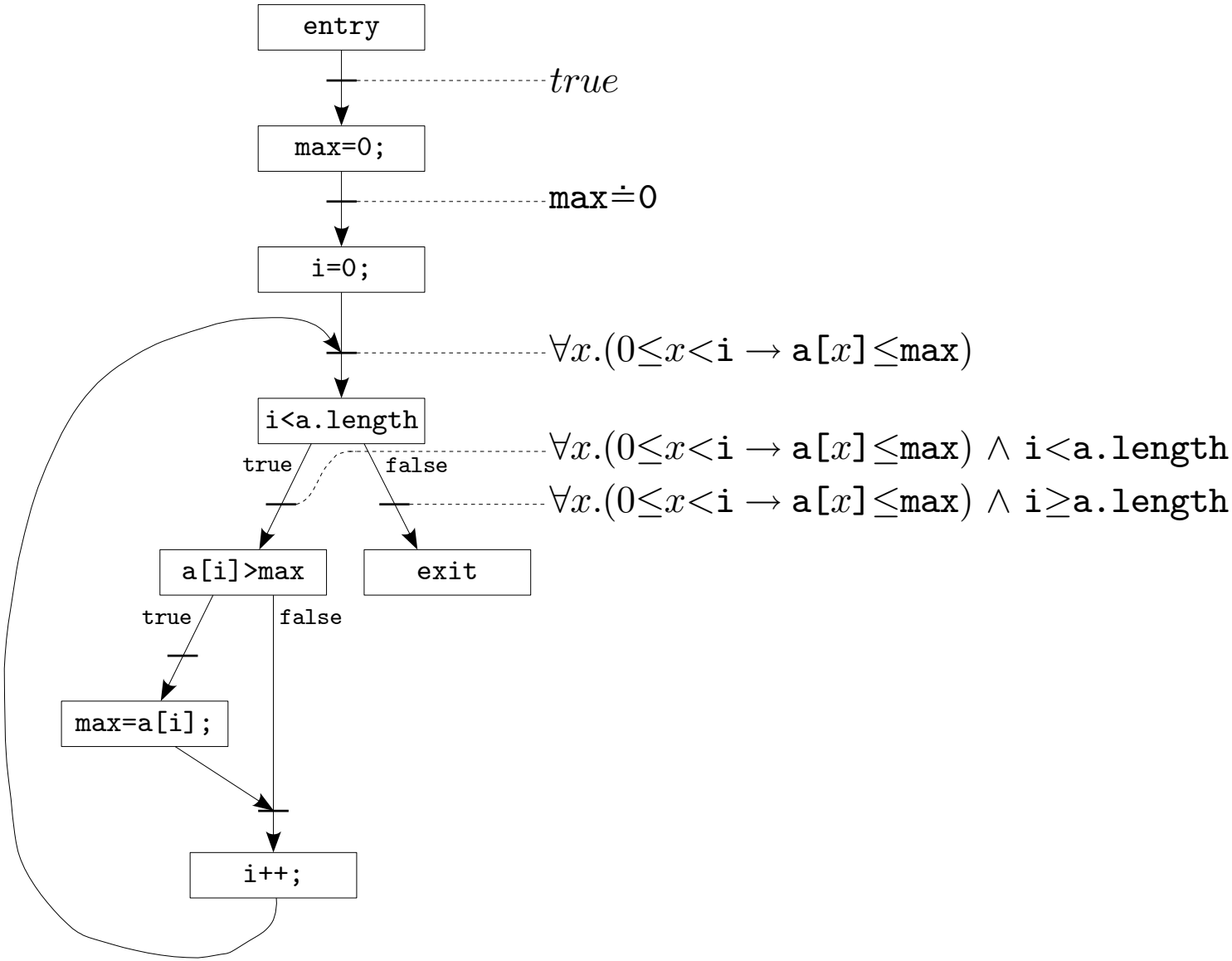

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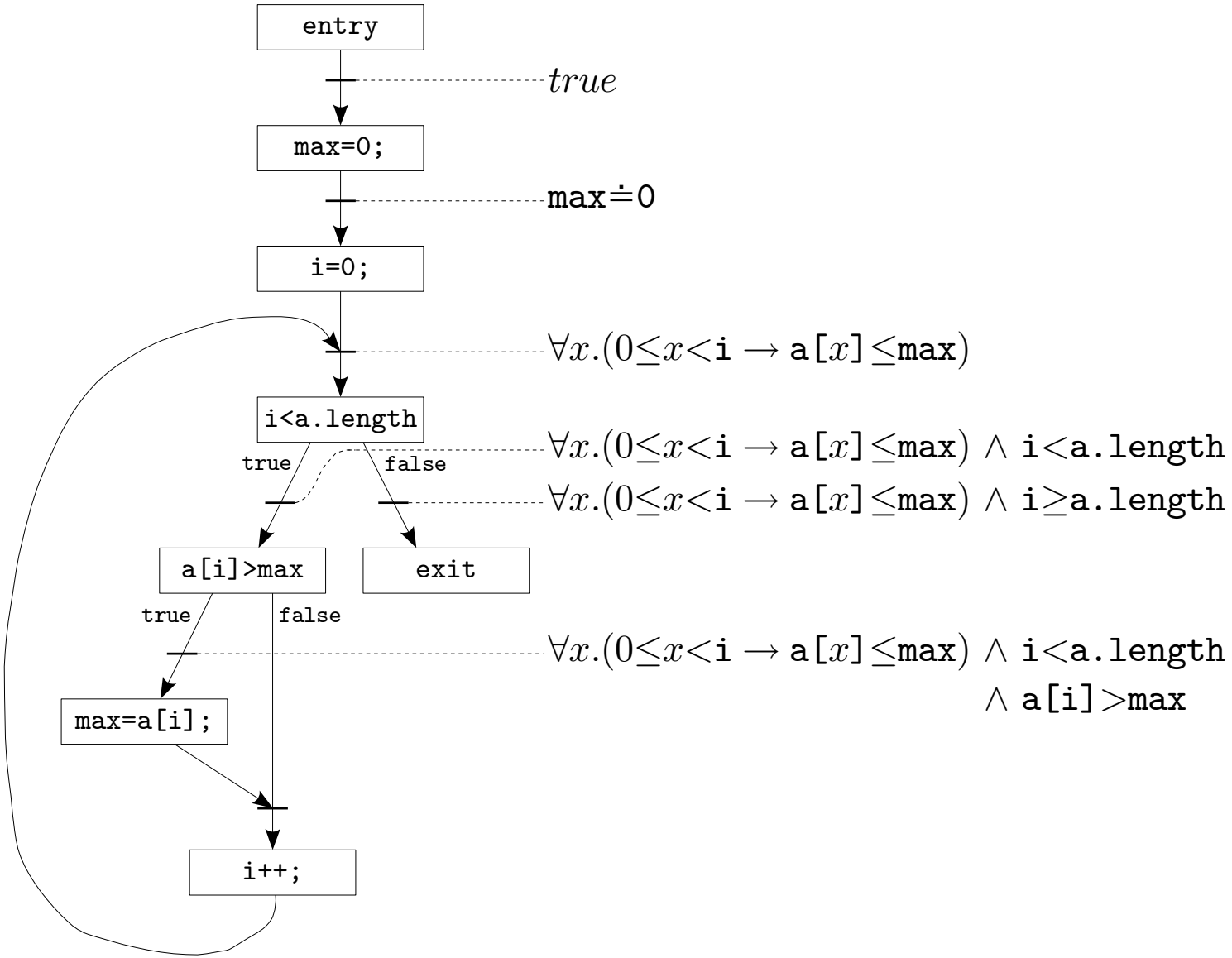

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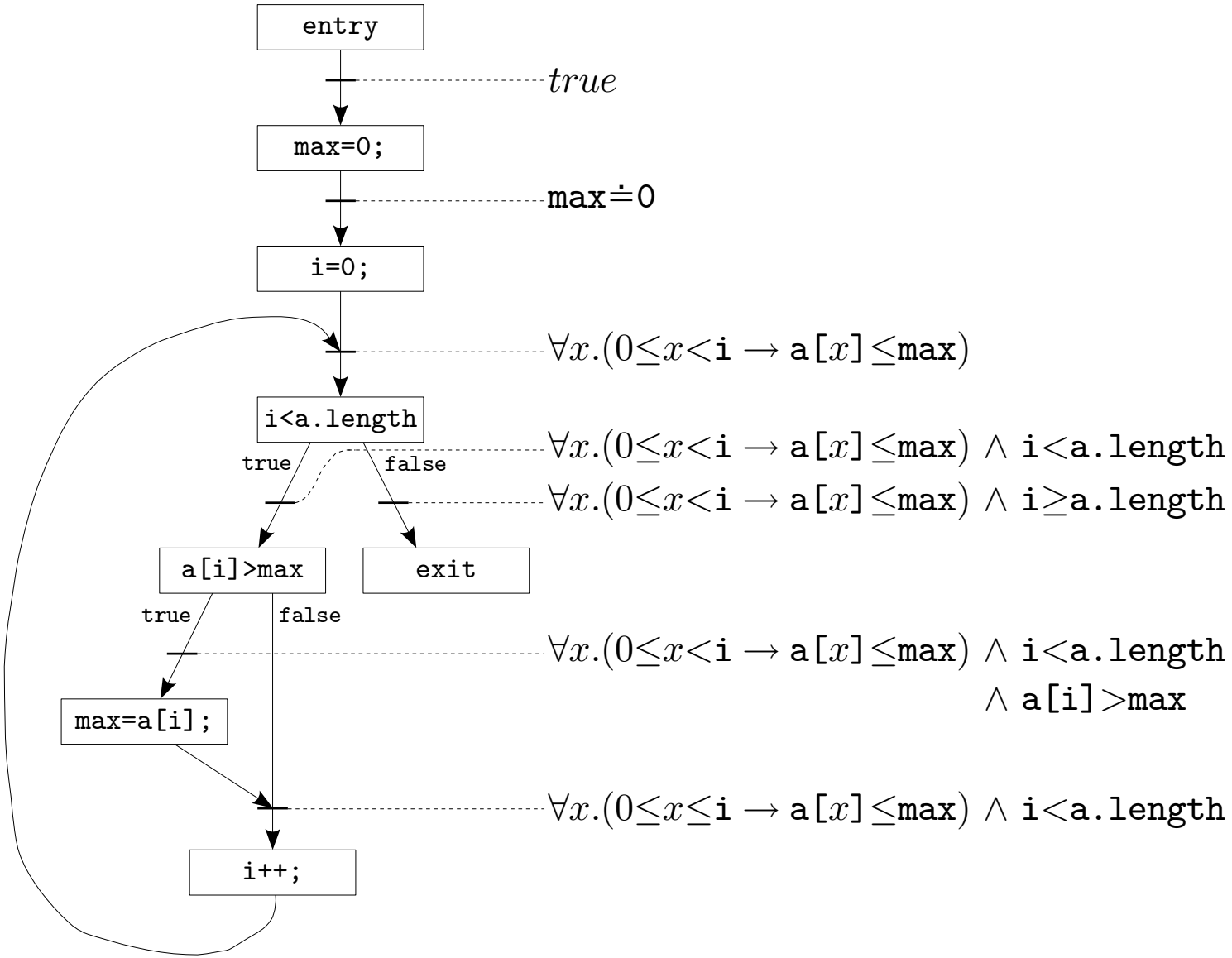

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An analysis which includes executing the program on concrete input values. (e.g. testing, Daikon)

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Static Analysis:

An analysis which does *not* actually execute the program. (e.g. KeY, abstract interpretation)

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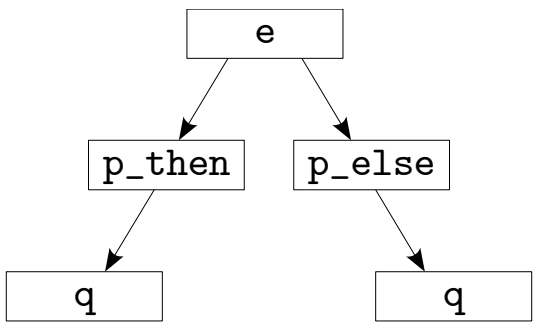
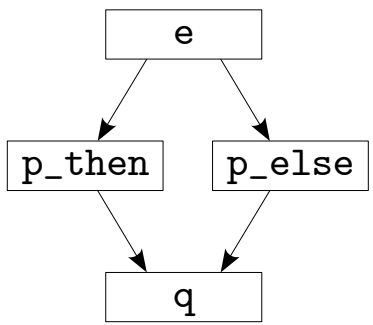


KeY

vs.



abstract interpretation

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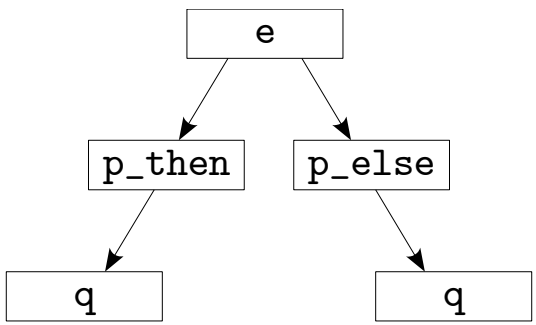

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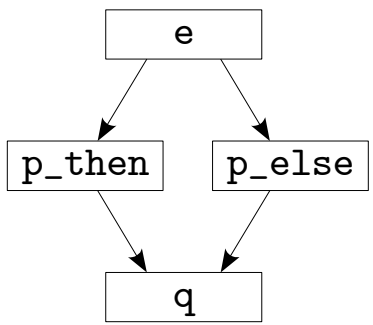

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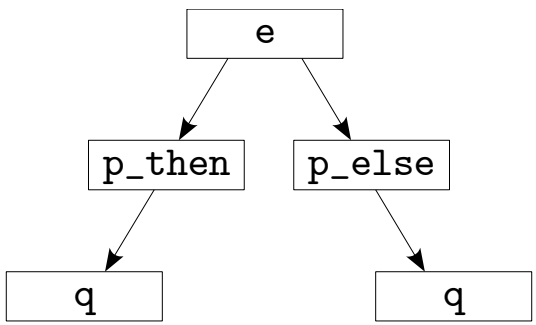

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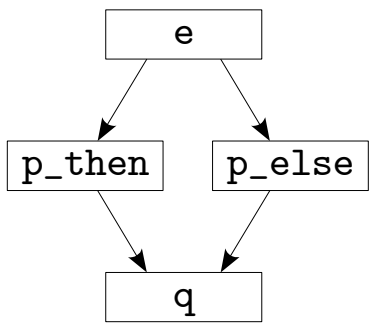

abstract interpretation

- Approximation
- Fixed-point iteration

Invariant Inference in KeY

$$\mathit{true} \longrightarrow [\ \texttt{max = 0;}\ \texttt{i = 0;}\ \texttt{while(i < a.length) \{...\}}\ ]\psi$$

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$$\varphi_1 \wedge \texttt{i} < \texttt{a.length} \rightarrow [\ \texttt{if(a[i] > max) max = a[i];}\ \texttt{i++;}\ \texttt{while(i < a.length) \{...\}}\ ]\psi$$

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$$\varphi_1 \wedge \texttt{i} \geq \texttt{a.length} \rightarrow []\psi$$

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- In both cases:
  - A few predicates were added by hand ($\rightsquigarrow$ quantified invariants)
  - Inferred invariants strong enough for verification

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Future work

- More experiments
- Improve implementation
- Establish soundness of all of the new rules
- Incorporate "counterexample-guided abstraction refinement" techniques (?)