Dual Pivot Quicksort: Verification and Proof using KeY

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Introduction

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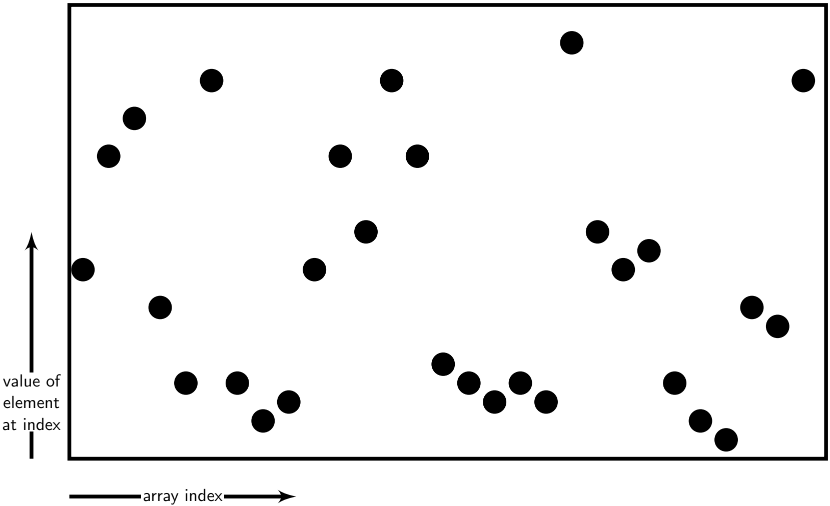
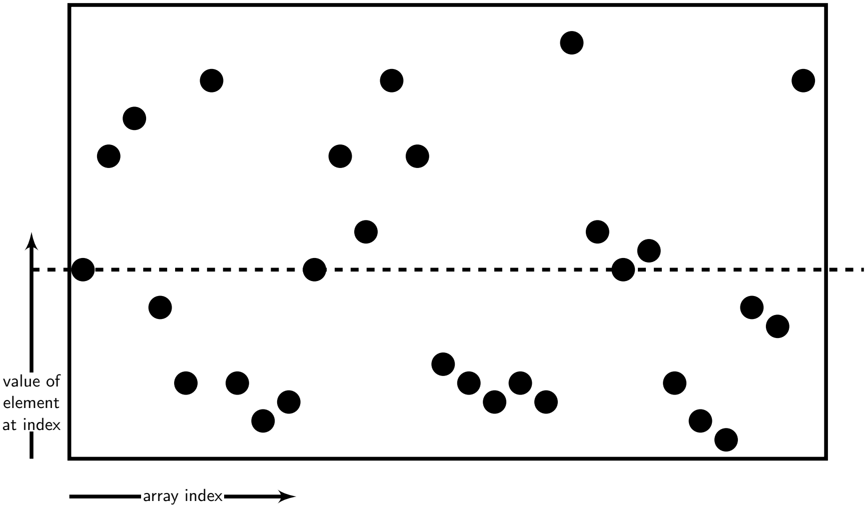
Section 1

Algorithm Description

Quicksort



Quicksort



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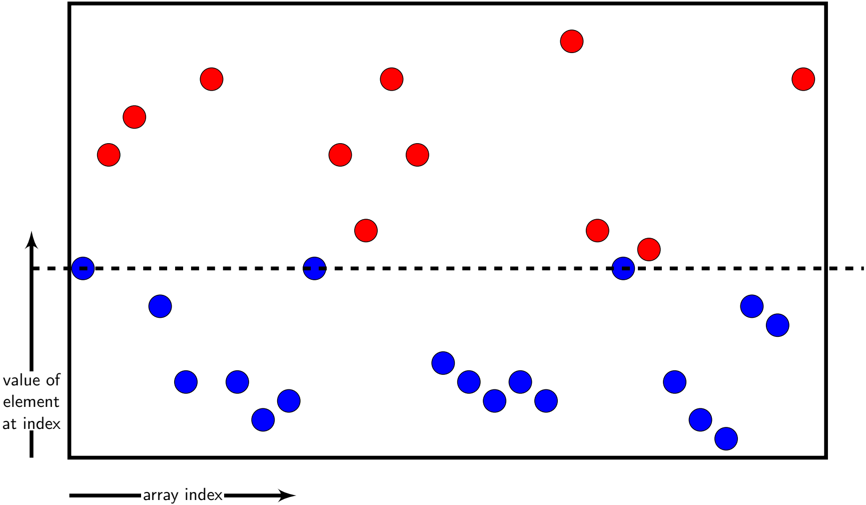

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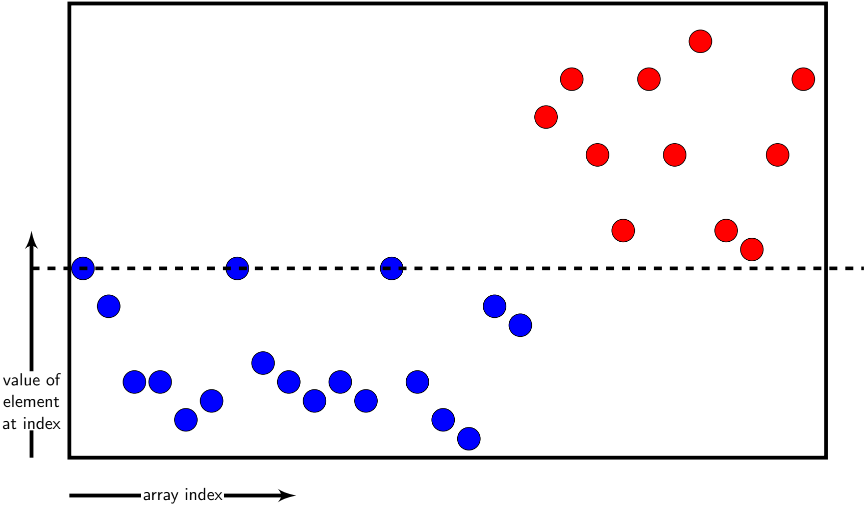

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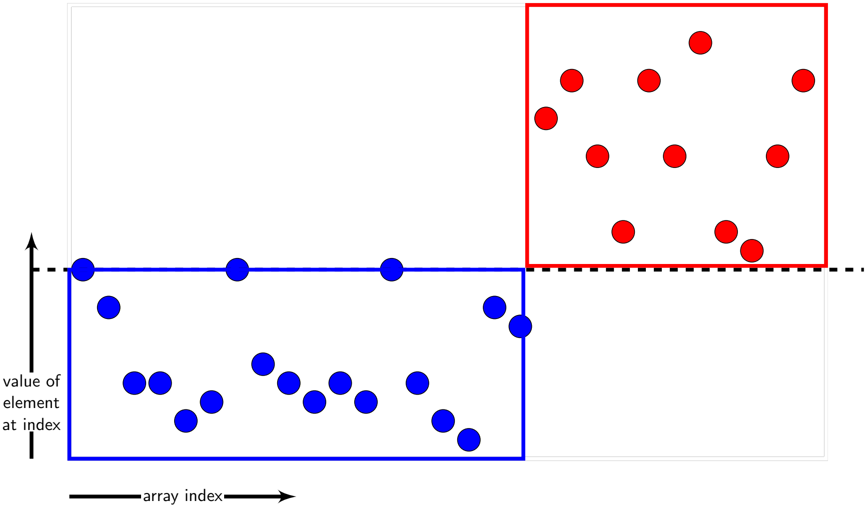

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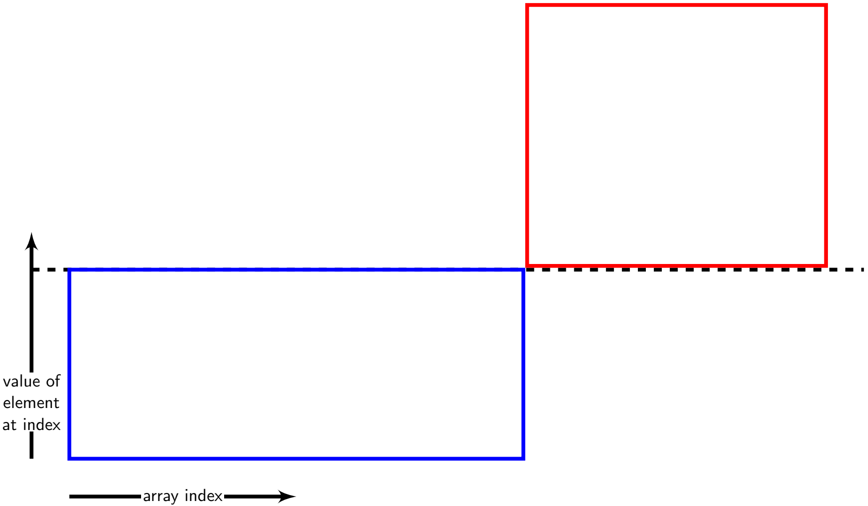

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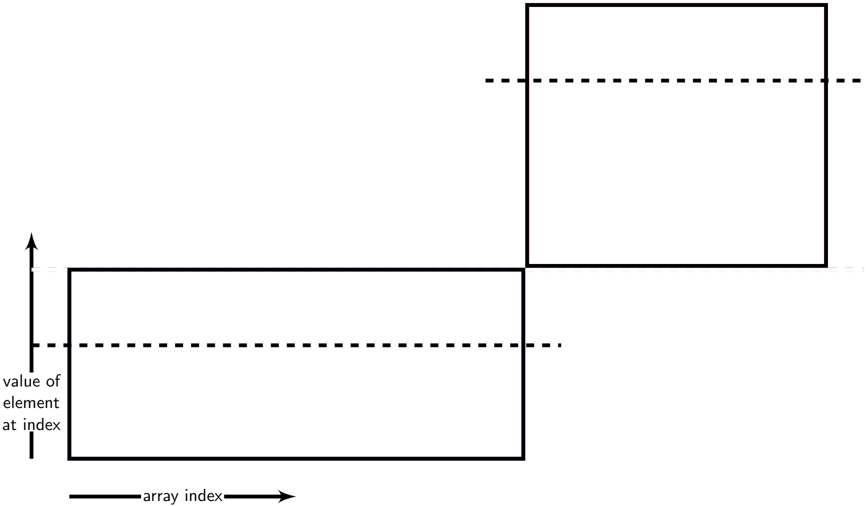

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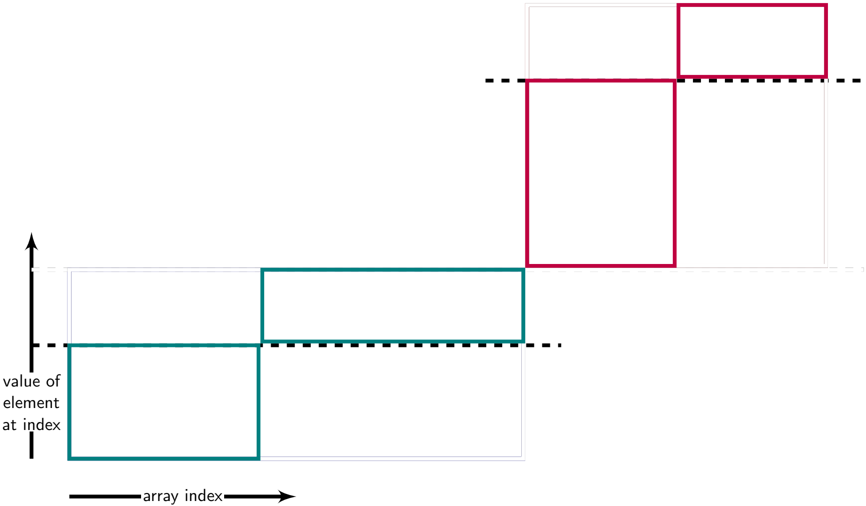

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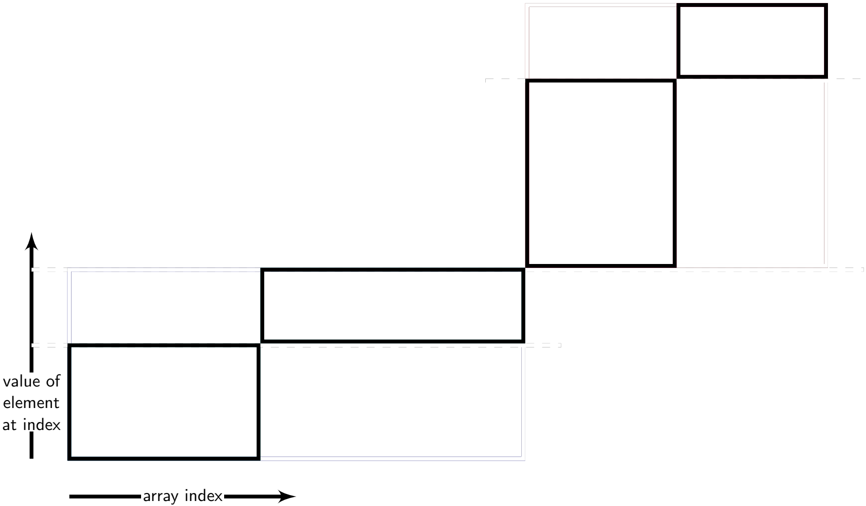
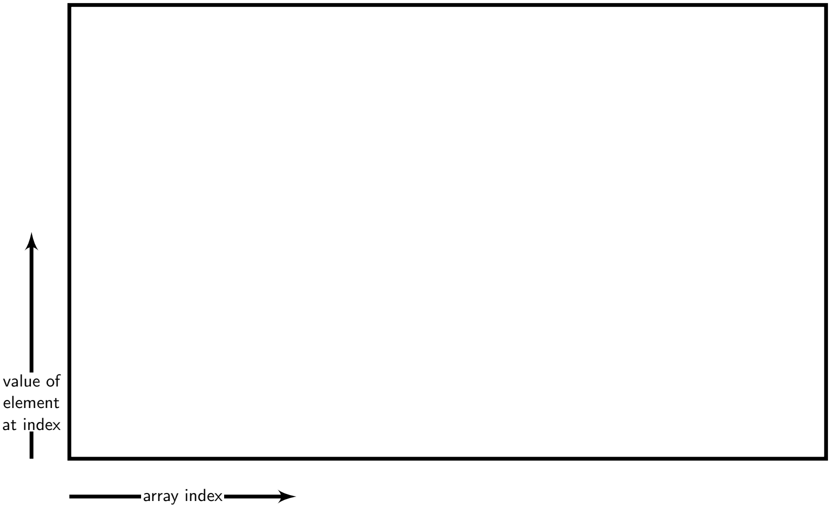


Dual Pivot Quicksort



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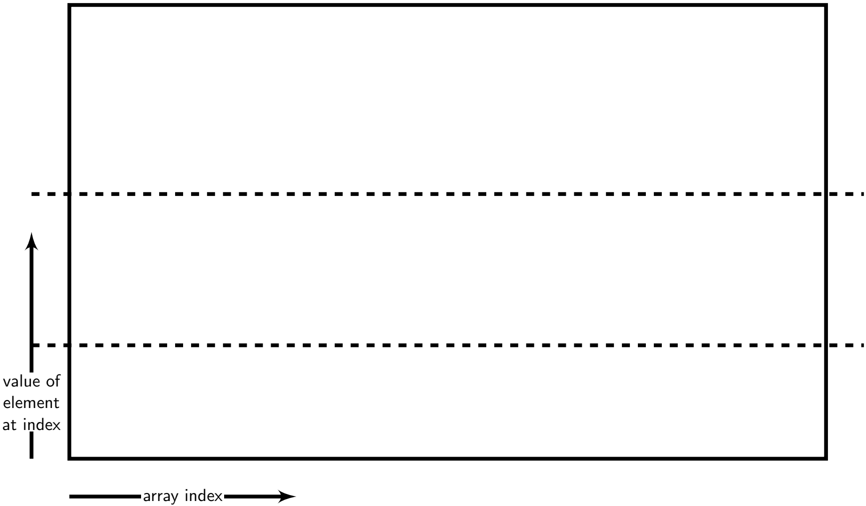

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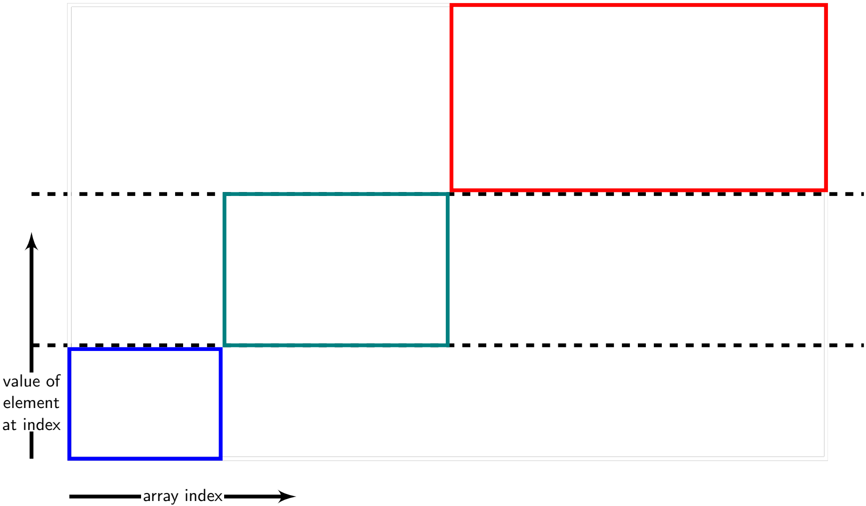

Dual Pivot Quicksort

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Why use Dual Pivot Quicksort?

- Theory: Average number of swaps reduced by 20% (Yaroslavskiy 2009)
- Practice: Multi-pivot Quicksorts are more cache-efficient (Kushagra 2014)
- Benchmarking shows it is faster

Java Implementation – Choosing a Sorting Algorithm

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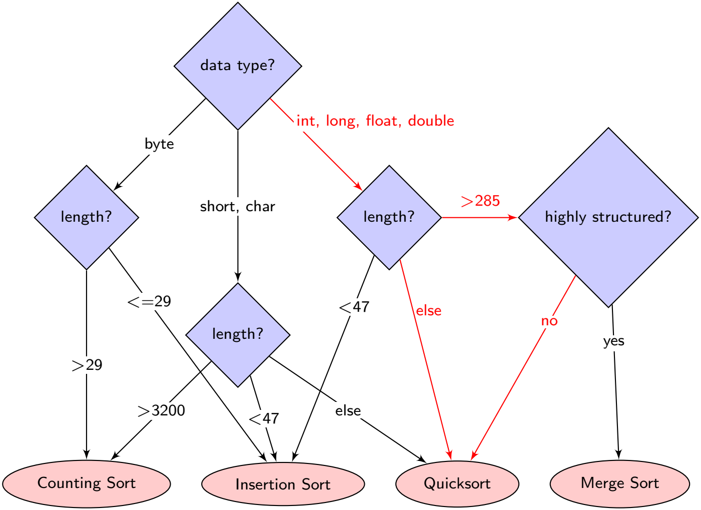

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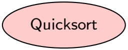

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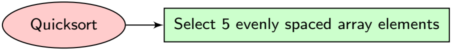

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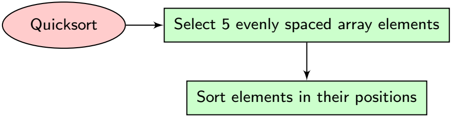
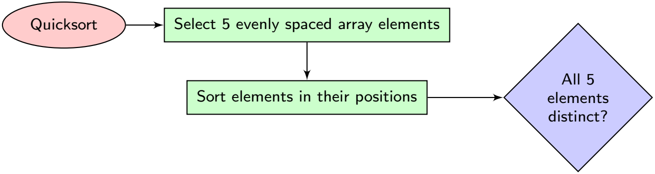


Quicksort

Select 5 evenly spaced array elements

Sort elements in their positions

Java Implementation – Quicksort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Select 5 evenly spaced array elements
Sort elements in their positions
All 5 elements distinct?

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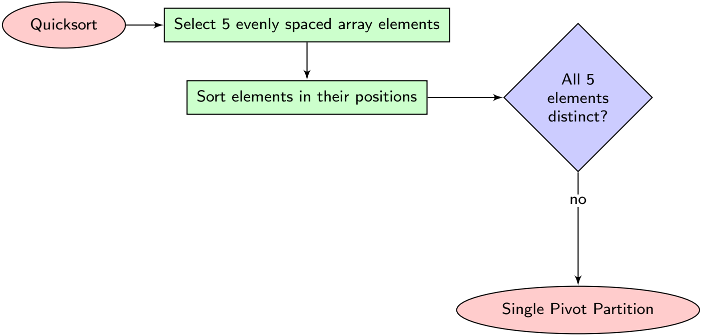
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Single Pivot Partition

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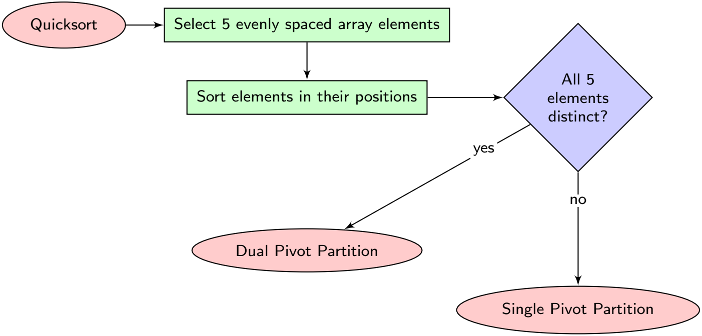

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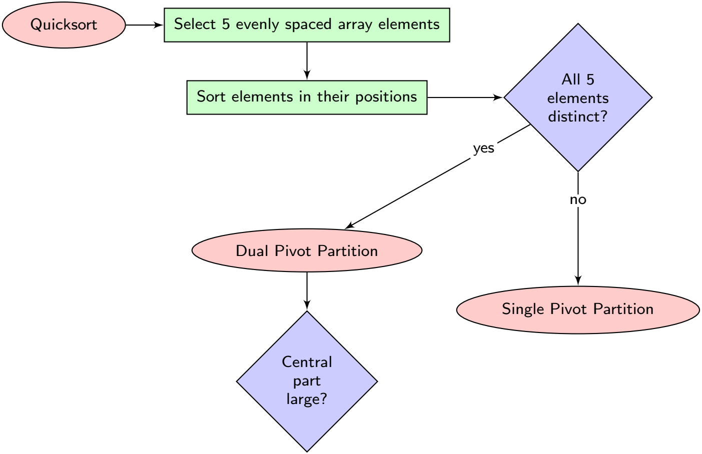

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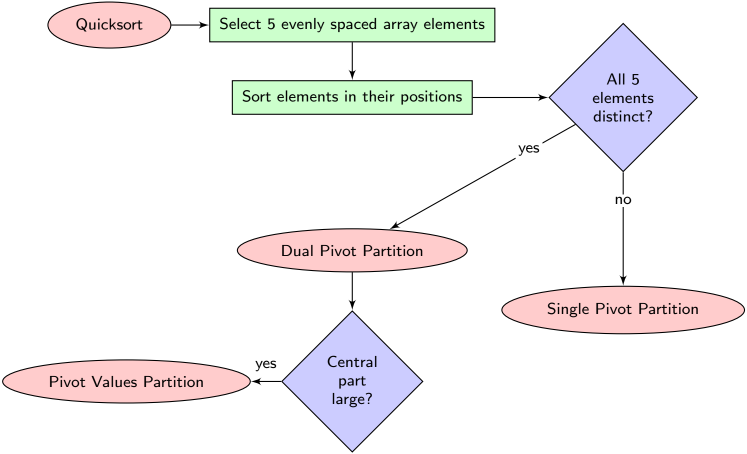

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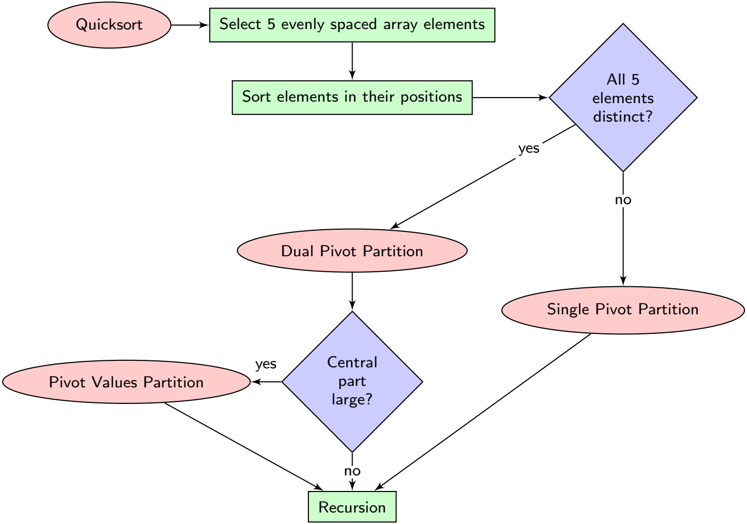
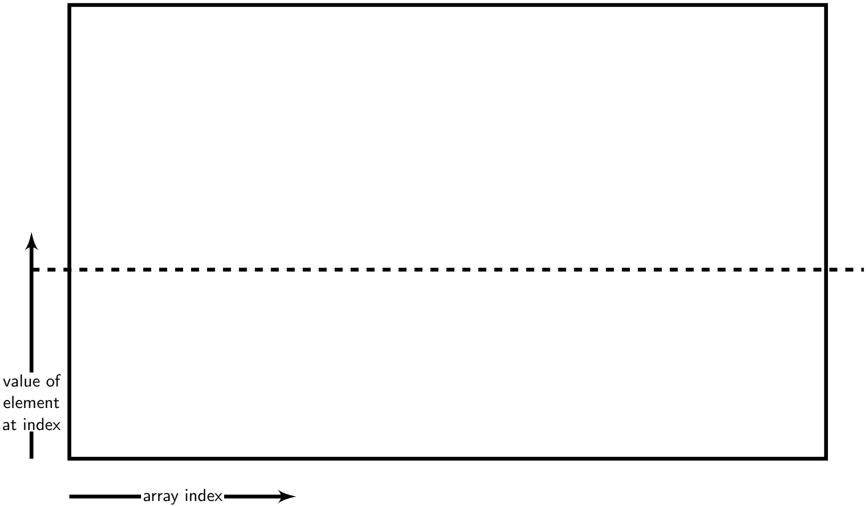


Java Implementation – Single Pivot Partition



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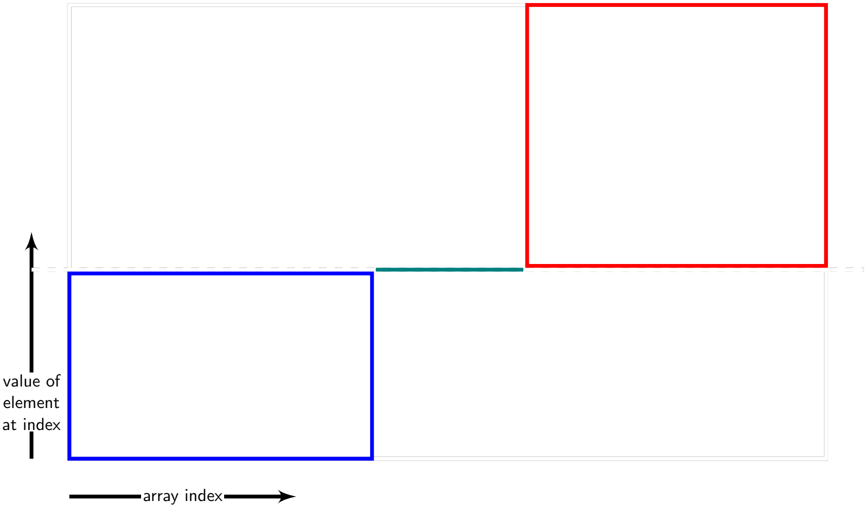

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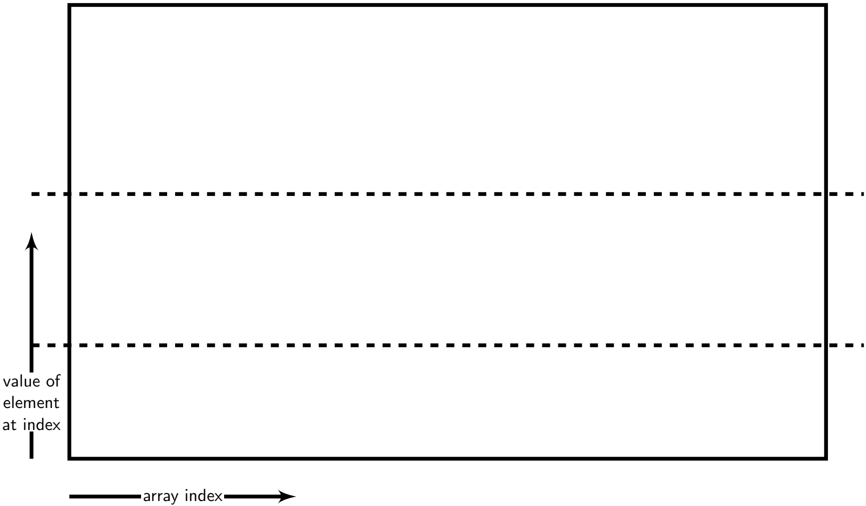

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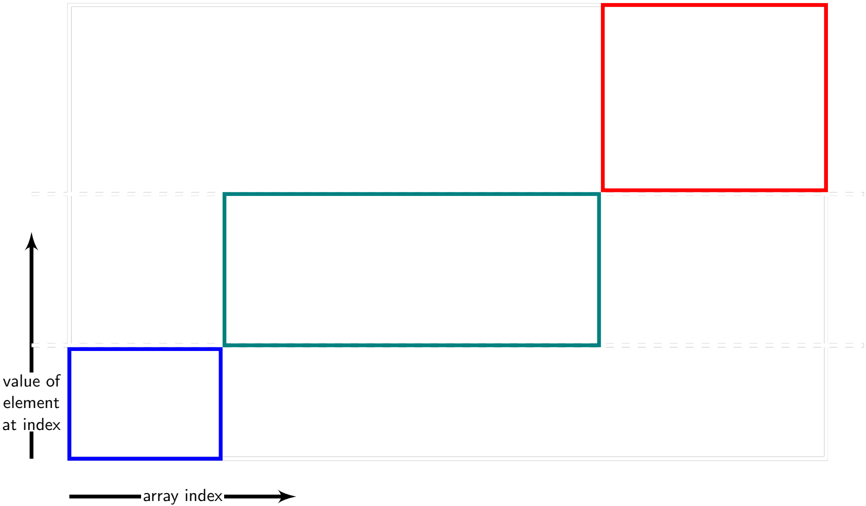
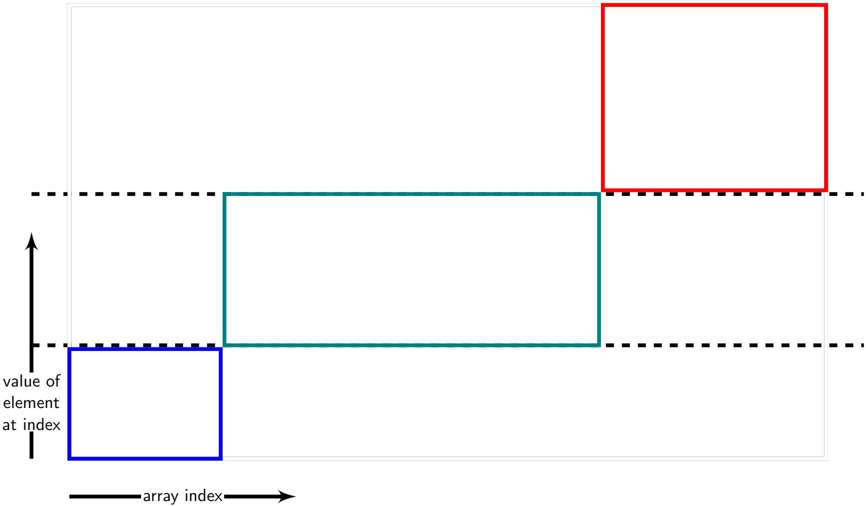


Java Implementation – Swap Pivot Values Partition



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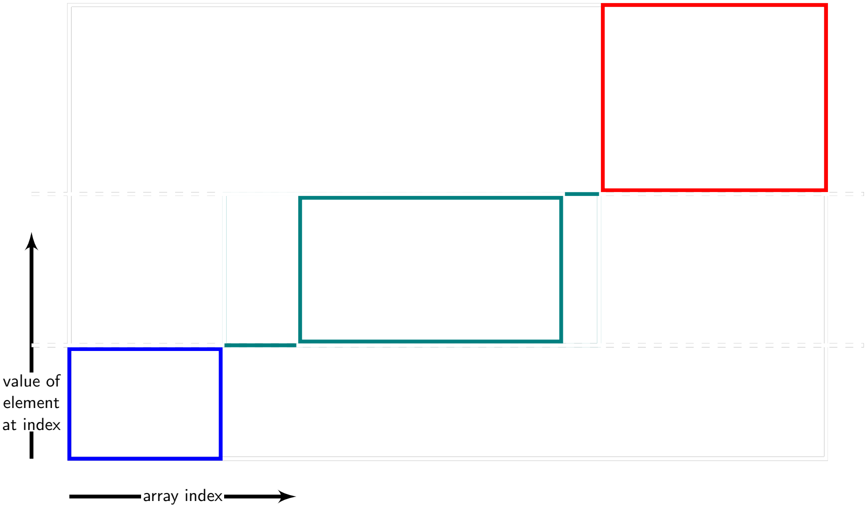
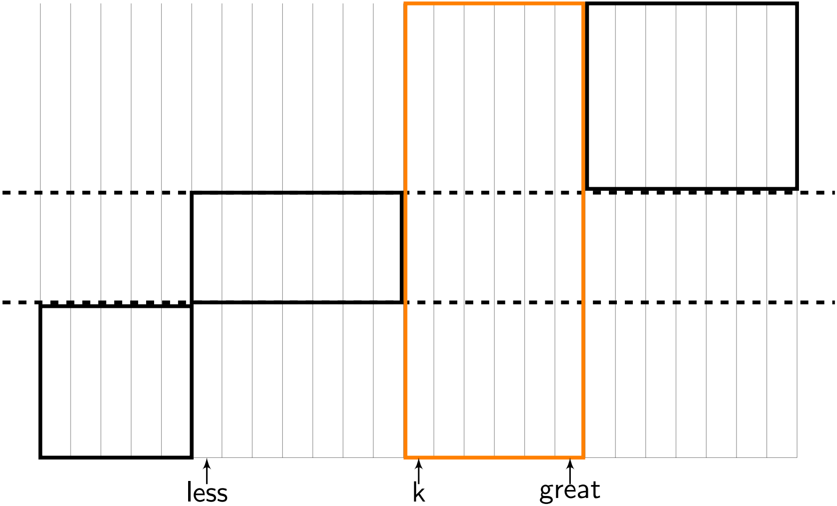


Java Implementation – Partitioning



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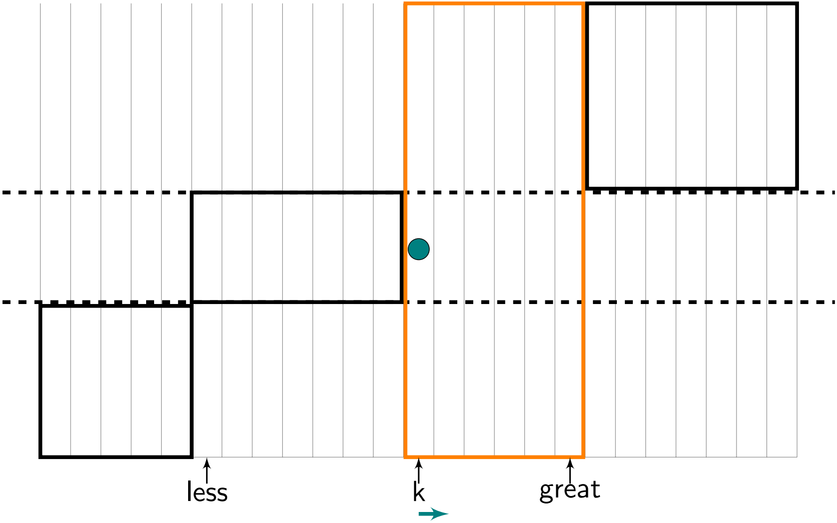

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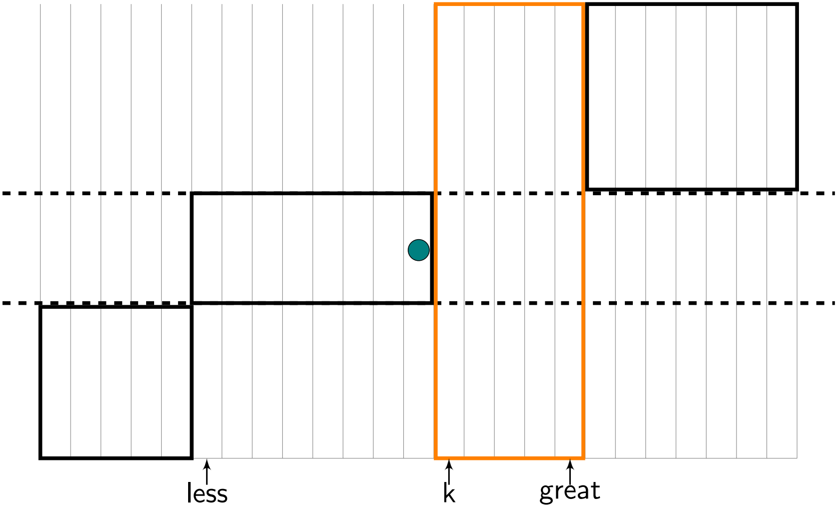

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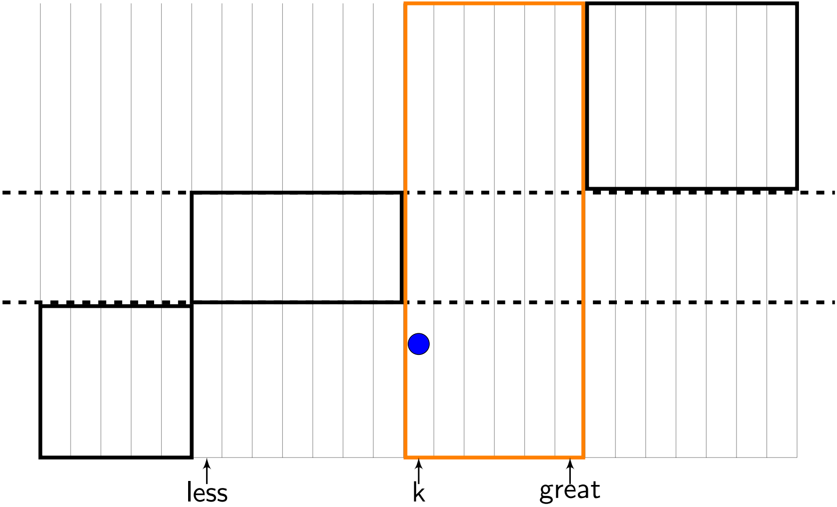

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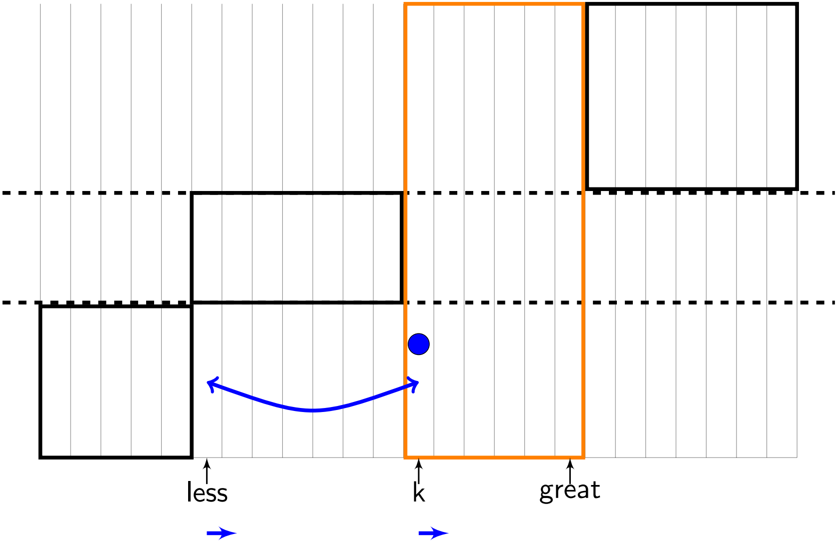

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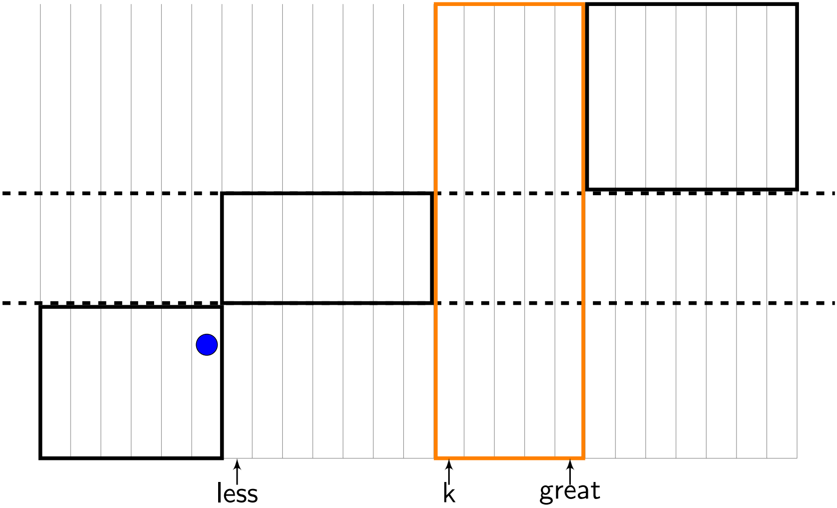

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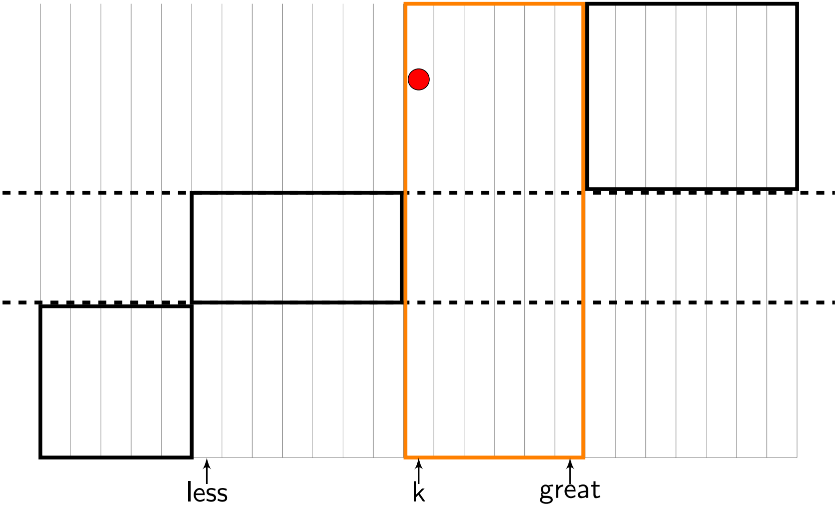

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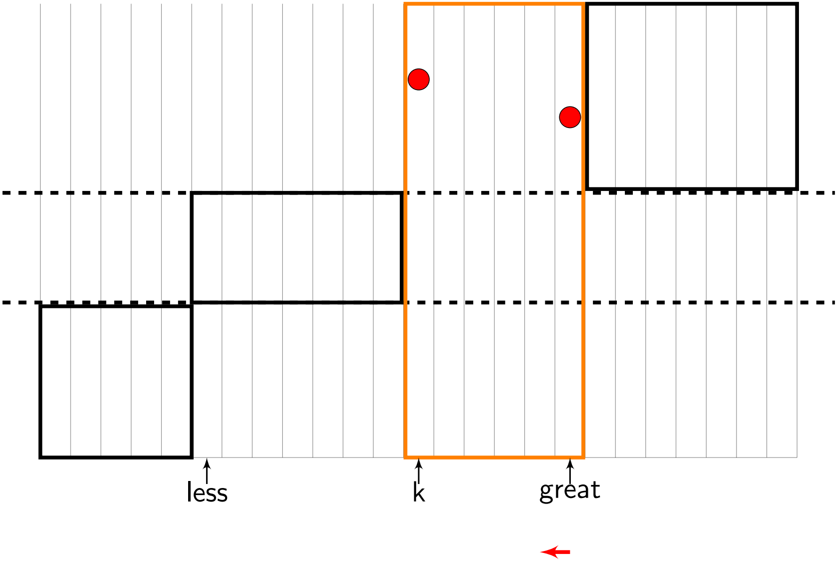

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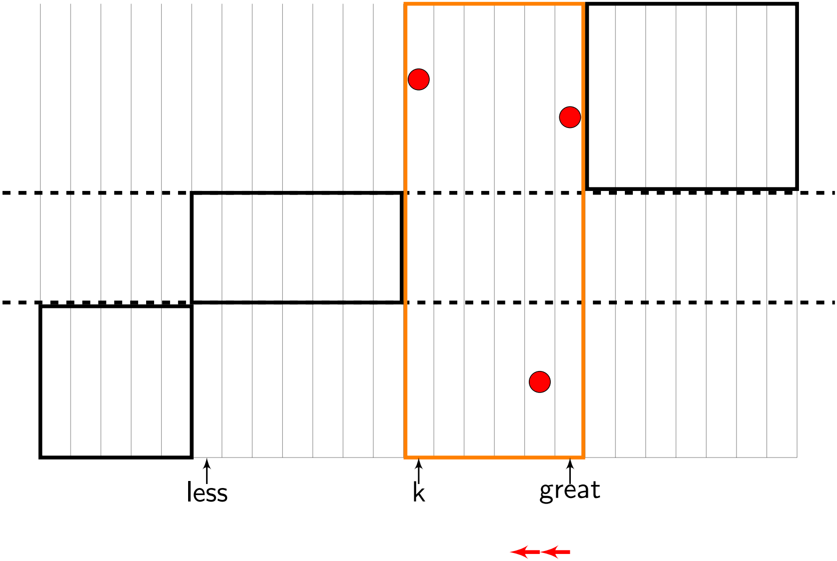

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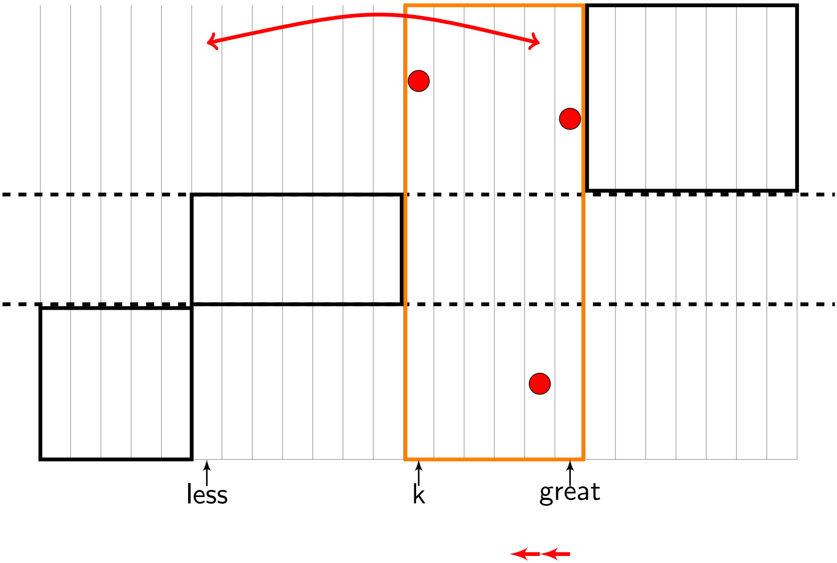

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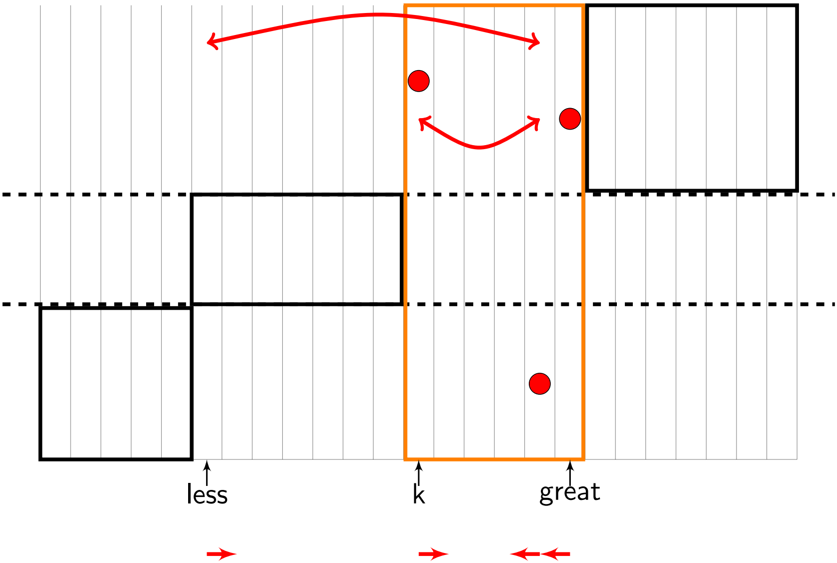

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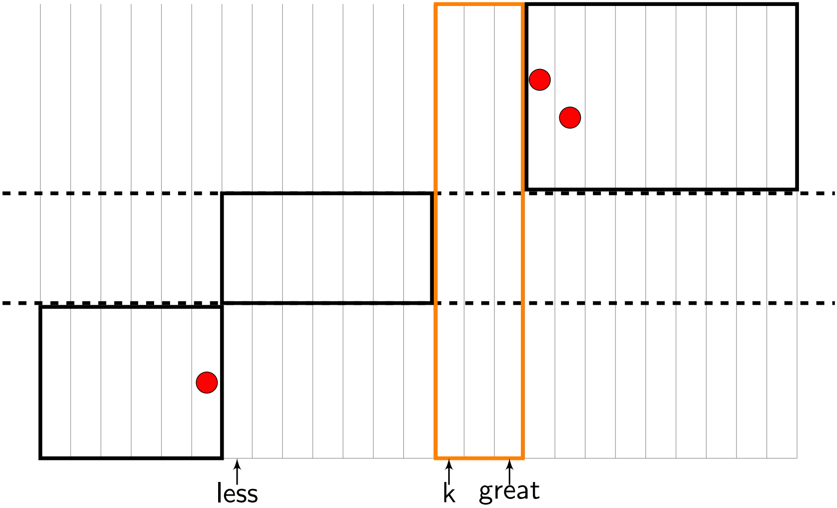

Section 2

Specification and Proof

Work Flow

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- Encapsulating source code in its own Java class

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- Writing specification
  Running KeY
  Adapting specification or source code

General KeY Strategy

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- If proof fails:
  - Confirm by generating counterexample
  - Find violated specification condition
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General KeY Strategy

- Autopilot Strategy Macro
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  - Adapt specification (or source code)
- If no proof is found:
  - Increase number of steps (?)
  - Interactive Rule Apps (Quantifier Instantiation, if-then-else-split)
  - Heap Simplification + SMT Solver

Feasibility – Problems with KeY

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- Computation time

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- Responsiveness

Violation of Single Pivot Partition Invariant

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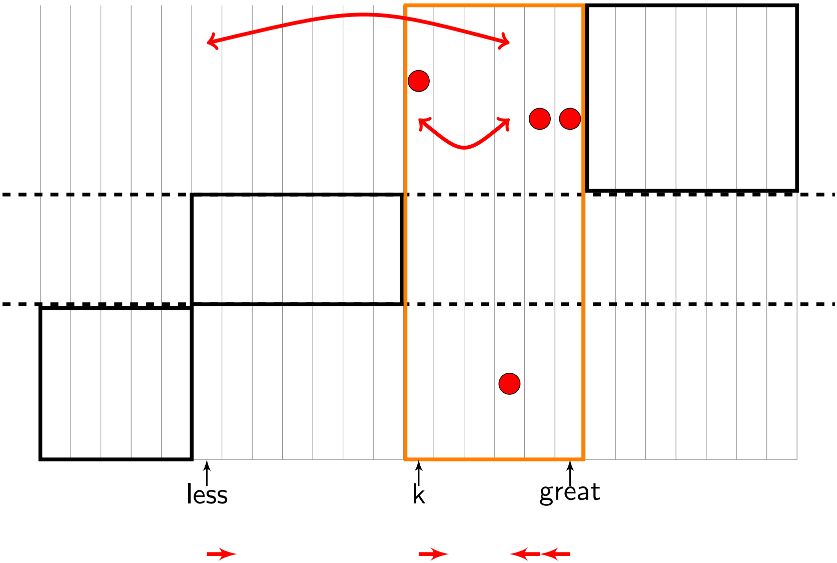


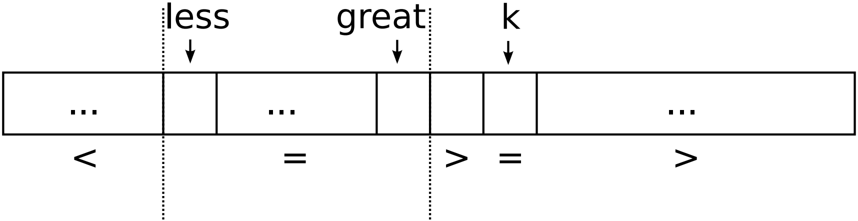
Violation of Single Pivot Partition Invariant

```
while (a[great] > pivot2) {
    if (great-- == k) {
        break outer;
    }
}

while (a[great] == pivot2) {
    if (great-- == k) {
        break outer;
    }
}

while (a[great] > pivot) {
    --great;
}
...
```

Violation of Single Pivot Partition Invariant



Section 3

Conclusive Remarks

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Statistics – Single Pivot Partition

Method	Nodes	Branches	Time [s]	Rule Apps	Interactive	SMT
case_right	14784	114	17,7	18919	0	0
split	17609	90	23,8	24189	0	0
sort(array, left, right)	18495	101	18,8	22839	0	0
sort(array)	654	7	0,4	1342	0	0
Total	51542	312	60.7	67289	0	0

Statistics – Swap Pivot Values Partition

Method	Nodes	Branches	Time [s]	Rule Apps	Interactive	SMT
move_great_left	1245	16	0,8	2346	0	0
move_less_right	2120	14	1,8	3224	0	0
swap_values	123636	407	246,6	138039	0	0
Total	127001	437	249.2	143609	0	0

Statistics – Dual Pivot Partition

Method	Nodes	Branches	Time [s]	Rule Apps	Interactive	SMT
calc_indices	24533	8	49,6	24835	0	0
insertionsort_indices	50816	365	137,4	73056	0	34
prepare_indices	5332	28	6,4	7153	0	0
move_great_left	1650	15	1,1	2605	0	0
move_great_in_loop	1580	18	1,1	2787	0	0
move_less_right	1928	14	1,4	2967	0	0
loop_body	52134	287	57,3	56263	18	0
split	28751	98	109,6	51666	0	36
sort(int[],left,right)	51342	305	459,6	76973	114	116
sort(int[])	611	5	0,4	1236	0	0
Total	218677	1143	823,9	299541	132	186
Entire Proof	297220	1892	1133,8	510439	132	186