Automated Verification for Functional and Relational Properties of Voting Rules

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Motivation: An Example

Exemplary election for candidates A, B, and C, and nine voters

Ballot Profile

Voter	Ballot
1	A
2	A
3	A
4	A
5	B
6	B
7	B
8	C
9	C

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Ballot Profile

Voter	Ballot
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2	A
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What should be the election outcome?

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1	A
2	A
3	A
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5	B , C
6	B , C
7	B , C
8	C
9	C

What should be the election outcome?

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Ballot Profile

Voter	Ballot
1	A > B > C
2	A > B > C
3	A > B > C
4	A > B > C
5	B > C > A
6	B > C > A
7	B > C > A
8	C > B > A
9	C > B > A

What should be the election outcome?

Motivation: An Example

Exemplary election for candidates A, B, and C, and nine voters

Ballot Profile

Voter	Ballot
1	A > **B** > C
2	A > **B** > C
3	A > **B** > C
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8	C > **B** > A
9	C > **B** > A

What should be the election outcome?
Candidate B?

Motivation: An Example

Exemplary election for candidates A, B, C, D, and E, and nine voters

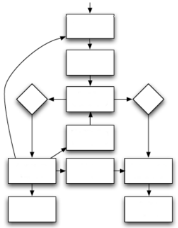
Ballot Profile

Voter	Ballot
1	A > **B** > **D** > **E** > C
2	A > **E** > **D** > **B** > C
3	A > **B** > **E** > **D** > C
4	A > **D** > **B** > **E** > C
5	**B** > **E** > **D** > C > A
6	**E** > **D** > **B** > C > A
7	**B** > **D** > **E** > C > A
8	C > **E** > **D** > **B** > A
9	C > **E** > **B** > **D** > A

What should be the election outcome?
Candidate B?
What if B is actually a coalition of the three candidates B, D, and E?

Motivation: The General Idea

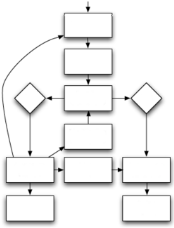
Voting Rule *V*



Motivation: The General Idea

Voting Rule *V*

Ballot Profile *B*



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Voting Rule *V*

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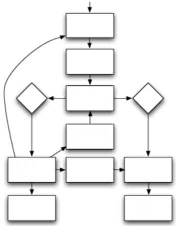
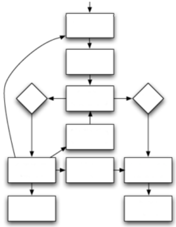


Outcome
V(B)

Motivation: The General Idea

Voting Rule V

Ballot Profile B



Outcome
$V(B)$

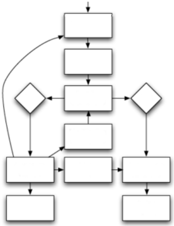
Axiomatic Property P

$$\forall x, y. \exists z \ldots$$

Motivation: The General Idea

Voting Rule *V*

Ballot Profile *B*



Outcome
V(B)

Axiomatic Property *P*

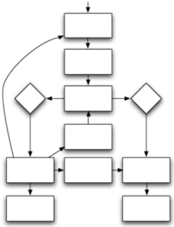
$\forall x, y. \exists z \ldots$

Does *V* satisfy *P*?

Motivation: The General Idea

Voting Rule *V*

Ballot Profile *B*



Outcome

V(B)

Axiomatic Property *P*

$\forall x, y.\exists z \ldots$

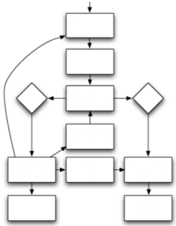
Does *V* satisfy *P*?

- Tedious, non-trivial and error-prone
- Especially for multiple properties
- Can this be automated?

Motivation: The General Idea

Voting Rule *V*

Ballot Profile B



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$V(B)$

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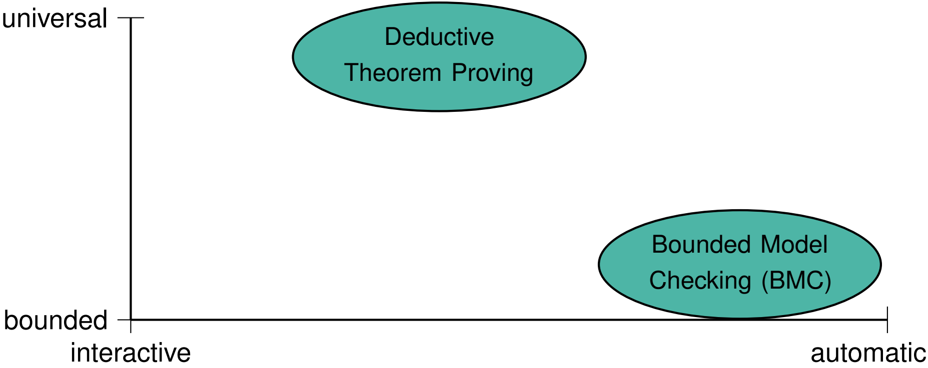
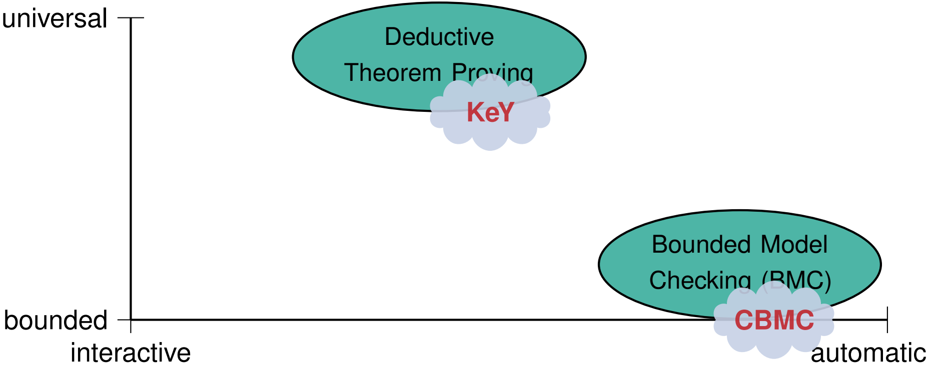
Computer-aided verification
for **trustworthy** voting rules!

Used Verification Techniques



Used Verification Techniques



- *Established* verification techniques

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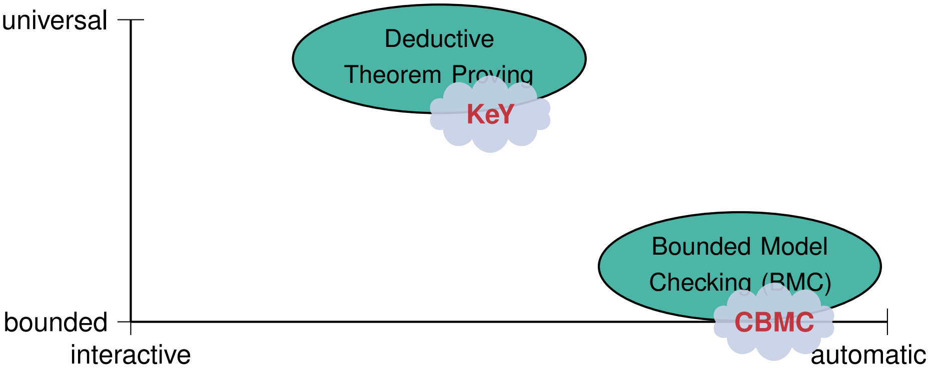

- *Established* verification techniques
- *Expressive* languages for imperative algorithms (C / Java) and properties ($FOL_{\mathbb{N}}$)

Functional and Relational Properties

Functional Properties (intra-profile (Fishburn 1973))

- Consider individual election evaluations (one profile with outcome)
- Examples: **majority criterion**, Condorcet criterion

Relational Properties (inter-profile (Fishburn 1973))

- Consider multiple election evaluations (two profiles with outcomes)
- Examples: **anonymity property**, monotonicity property

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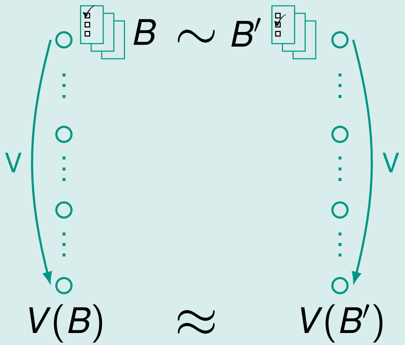
Verification of Relational Properties

Separate Evaluations



Verification of Relational Properties

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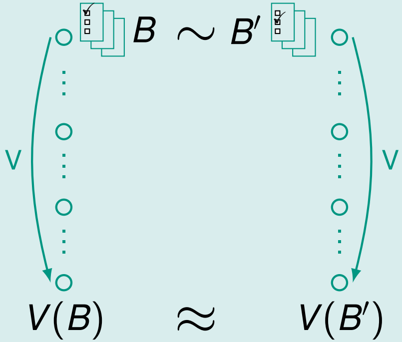
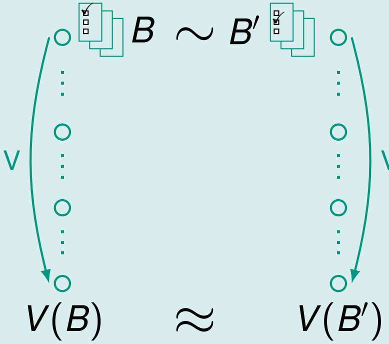


Example

$$\max_c \sum_{i=0}^{N} B_{i,c} = \max_c \sum_{i=0}^{N} B'_{i,c}$$

Verification of Relational Properties

Separate Evaluations



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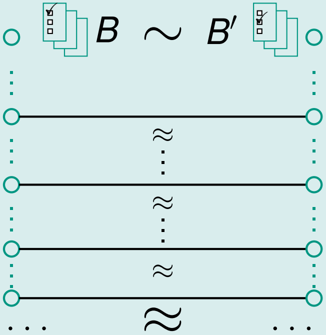
Coupling Evaluations



Verification of Relational Properties

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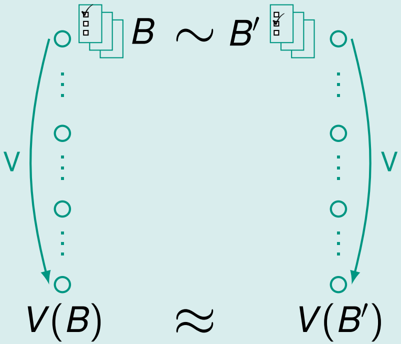

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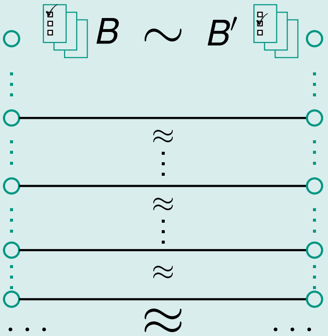
Coupling Evaluations



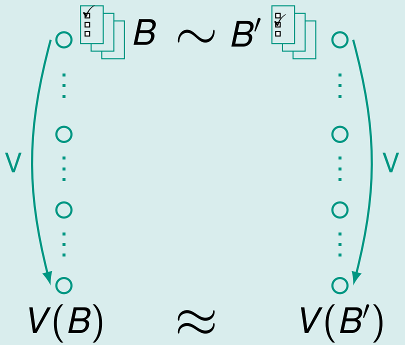
Example

result1 = *result2*

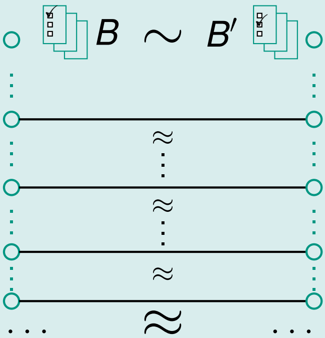
Verification of Relational Properties

Karlsruhe Institute of Technology

Separate Evaluations



Coupling Evaluations



Relational Verification

- Often enables short and concise specifications (only **differences**)
- Eases verification effort

Specification with Coupling Evaluations

Example: Homogeneity for plurality rule

V: Each voter chooses one candidate, candidate with most votes wins

P: Outcome only depends on **proportion** of each ballot type, i.e., if every ballot is replicated *N* times, the outcome is indifferent

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/*@ requires votes1.length == V ∧ votes2.length == N ∗ V;
  @ requires (∀ int a; 0 ≤ a < V; 0 ≤ votes1[a] < C);
  @ requires (∀ int a; 0 ≤ a < N∗V; 0 ≤ votes2[a] < C);
  @ requires (∀ int v,k; 0 ≤ v < V ∧ 0 ≤ k < N;
  @         votes1[v] == votes2[k + v ∗ N]);
  @ assignable res1, res2, result1, result2;
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```

Example: JML method contract for homogeneity

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`res1` and `res2`: arrays for counting the candidates' votes

Specification with Coupling Evaluations

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```

`result1` and `result2`: fields storing the elected candidates

Specification with Coupling Evaluations

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Wellformedness conditions

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```

Precondition for homogeneity

Coupling Loop Invariants: Homogeneity

Example: Summing up individual votes into arrays

```
/*@ loop_invariant 0 ≤ i1 ≤ V ∧ i1 * N == i2
  @ ∧ (∀ int c; 0 ≤ c < C; res2[c] == N*res1[c]);
  @ assignable res1[*], res2[*];
  @ decreases V − i1;
  @*/
for (int i1 = 0, int i2 = 0; i1 < V || i2 < V * N;)
{     if (i1 < V) res1[votes1[i1++]]++;
      while (i2 < i1 * N) res2[votes2[i2++]]++;
}
```

Coupling Loop Invariants: Homogeneity

First evaluation: One single run

```
/*@ loop_invariant 0 ≤ i1 ≤ V ∧ i1 ∗ N == i2
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}
```

Coupling Loop Invariants: Homogeneity

Second evaluation: One run replicated N times

```
/*@ loop_invariant 0 ≤ i1 ≤ V ∧ i1 * N == i2
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}
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Coupling Loop Invariants: Homogeneity

Coupling invariant: Relationship between both arrays

```
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}
```

Coupling Loop Invariants: Homogeneity

Coupling evaluations: Loop invariant for replicated run

```
for (int i1 = 0, int i2 = 0; i1 < V || i2 < V * N;)
{   if (i1 < V) res1[votes1[i1++]]++;
    /*@ loop_invariant 0 < i1 ≤ V ∧ i2 ≤ votes2.length
      @ ∧ (i1 − 1) * N ≤ i2 ≤ i1 * N
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      @    == res1[votes1[i1 − 1]] * N + (i2 − i1 * N);
      @ assignable res2[*];
      @ decreases (i1 + 1) * N − i2;
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    while (i2 < i1 * N) res2[votes2[i2++]]++; }
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Coupling Loop Invariants: Homogeneity

Range restrictions

```
for (int i1 = 0, int i2 = 0; i1 < V || i2 < V * N;)
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```

Coupling Loop Invariants: Homogeneity

Framing invariant for results from previous rounds

```
for (int i1 = 0, int i2 = 0; i1 < V || i2 < V * N;)
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Coupling Loop Invariants: Homogeneity

Relationship for current round, not strictly necessary

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Coupling Loop Invariants: Homogeneity

Current result array relationship, $i1 * N$ is distance from "compartment" start

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Verification with Coupling Evaluations

Example: Verification using KeY (including required lines of specification)

	Plurality V.	Approval V.	Range V.	Borda Count
Anonymity	33	43	44	44
Neutrality	42	56	57	57
Monotonicity	46	47	48	52
Participation	28	50	51	50
Homogeneity	53	70	71	71

- Case study for multiple rules and properties
- Breaks down verification effort (roughly) to functional verification
- Verification using separate evaluations often not feasible
- Concise specifications also useful for bounded model checking
  $\rightarrow$ Guides solver to achieve higher bounds

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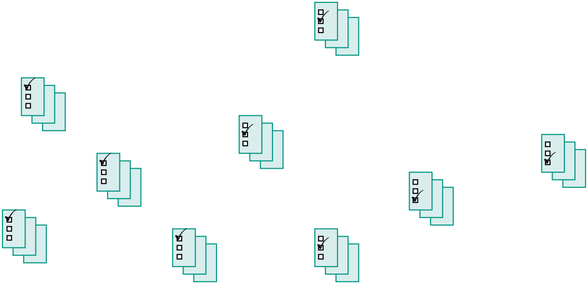
  $\rightarrow$ Guides solver to achieve higher bounds

Exploiting Symmetries on Functional Properties



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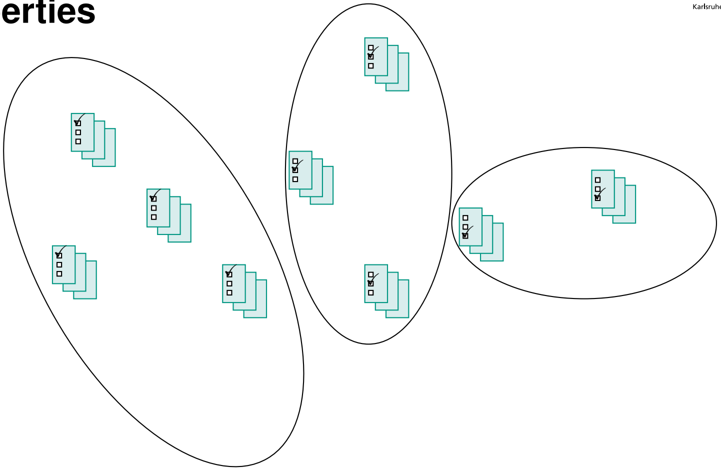
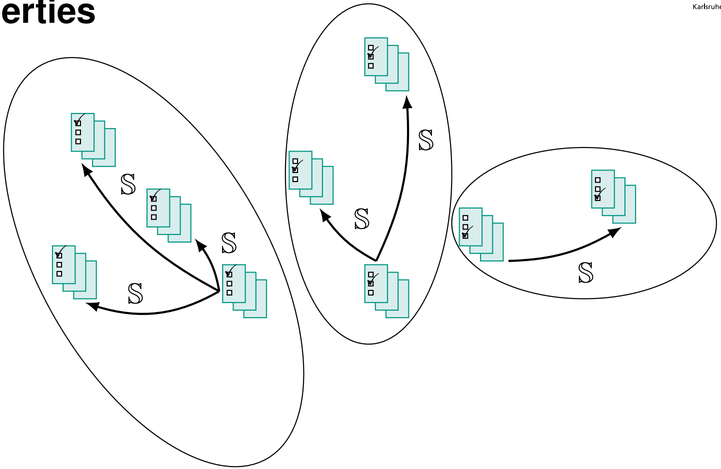


Symmetric profiles (for a symmetry property $\mathbb{S}$)

Exploiting Symmetries on Functional Properties



Symmetric profiles (for a symmetry property $\mathbb{S}$)
are reachable via symmetry (profile-) operations.

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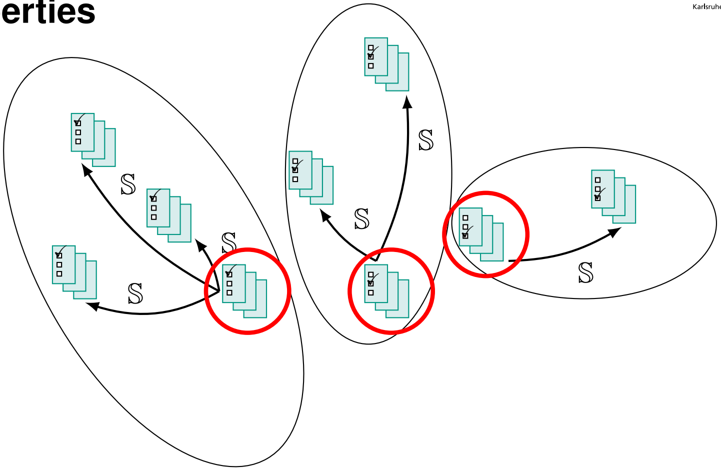
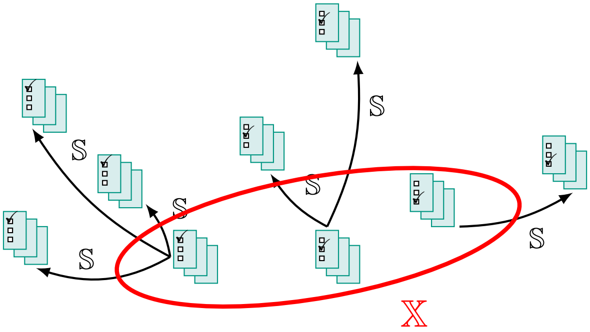


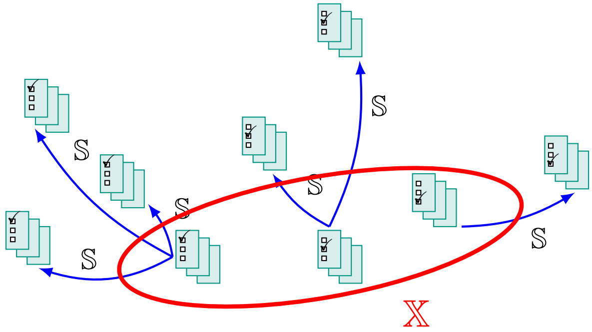
Symmetric profiles (for a symmetry property $\mathbb{S}$)
are reachable via symmetry (profile-) operations from minimal elements.

Exploiting Symmetries on Functional Properties



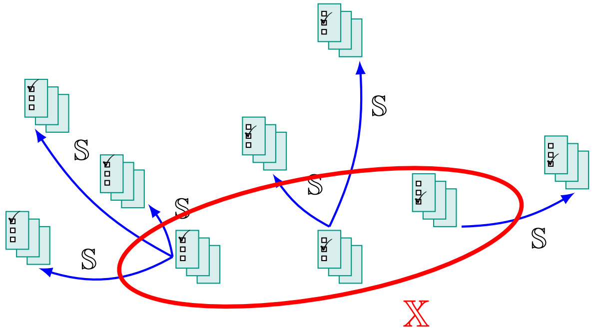
These minimal elements form a set $\mathbb{X}$,

Exploiting Symmetries on Functional Properties



These minimal elements form a set $\mathbb{X}$, via which *all possible profiles* are reachable.

Exploiting Symmetries on Functional Properties



Hence, if $\mathbb{S}$-operations preserve the desired property $\boldsymbol{P}$,

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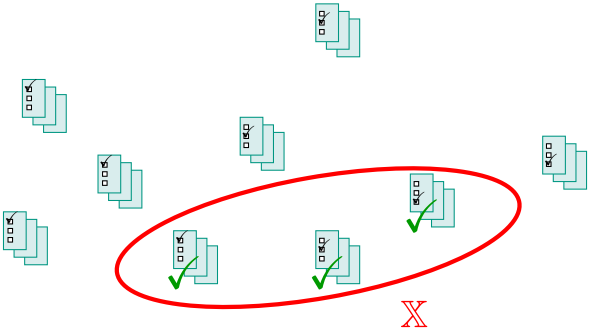

Hence, if $\mathbb{S}$-operations preserve the desired property ***P***,
verifying ***P*** only for elements in $\mathbb{X}$ is sufficient.

Verification of Functional Properties

- Verification Task: Does voting rule $\boldsymbol{V}$ satisfy property $\boldsymbol{P}$?
- Conjecture: $\boldsymbol{V}$ satisfies symmetry property $\mathbb{S}$.

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General Theorem for Verification

1. Verify $\mathbb{S}$ for $\boldsymbol{V}$ using relational techniques
2. Verify $\boldsymbol{V}$ satisfies property $\boldsymbol{P}$ only for subset $\mathbb{X}$
3. Prove that $\mathbb{X}$ spans *all possible profiles*
4. Prove that $\mathbb{S}$-operations preserve property $\boldsymbol{P}$

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Example

$\boldsymbol{V}$: Plurality rule

$\boldsymbol{P}$: Majority criterion

$\mathbb{S}$: Anonymity property

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$\mathbb{X}$: All sorted (by chosen candidate) profiles

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Example for anonymity property and plurality rule

- Profiles denoted as $(b_1, \ldots, b_N)$ (N number of cast ballots)
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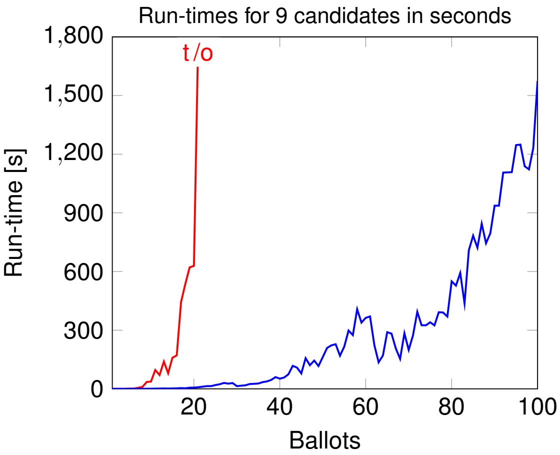
Example: Verification using bounded model checking (CBMC)



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- Case study for multiple rules and properties
- Composition of symmetries: anonymity **plus** neutrality

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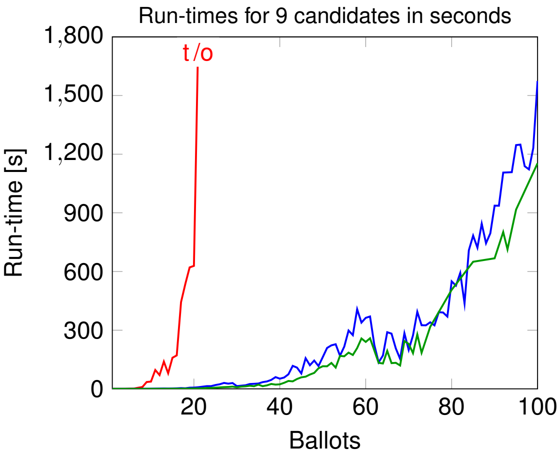

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Conclusion and Outlook

Karlsruhe Institute of Technology

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  $\Rightarrow$ Often critical point to make **verification feasible!**
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Questions and Answers

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Any questions?

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References

Peter C. Fishburn. *The Theory of Social Choice*. Princeton University Press, 1973 (cit. on pp. 18, 19).

Verification with Coupling Evaluations

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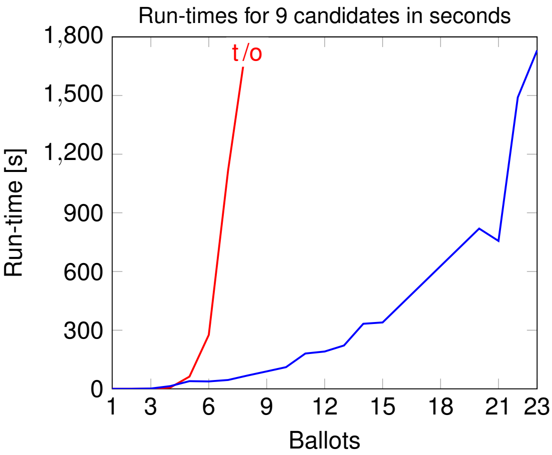

- Verified anonymity for plurality rule
- Concise specifications useable for BMC ⇒ Guidance for SAT-solver
- Separate and coupling evaluations
- Results: Achieved higher bounds