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Algorithmik I
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Local Ordered Covering Numbers

Background.

The *page number* of a graph $G = (V, E)$ is a graph parameter, denoted by $p(G)$, that is well-studied, for example in Graph Drawing. It is defined as the minimum k such that V can be linearly ordered and E can be k-colored in such a way that no two edges of the same color are crossing in the ordering $<$. Formally, for any two edges $ab, xy \in E$ we have:

$$a < x < b < y \quad \Rightarrow \quad \text{color}(ab) \neq \text{color}(xy) \tag{1}$$

Recently, a "local variant" of the page number has been introduced: the *local page number* of G, denoted by $p_L(G)$. Here we allow any numbers of colors but minimize the largest number of colors at any vertex rather than the total number of colors. Formally, $p_L(G)$ is the minimum ℓ such that V can be linearly ordered and E can be k-colored in such a way that (1) holds and

$$\#\{\text{color}(uv) \mid uv \in E\} \leq \ell \quad \text{for every vertex } v \in V. \tag{2}$$

Directions.

Include but are not limited to:

- Further develop the theory of local page numbers.
- Start the investigation of "local variants" of other ordered covering numbers, such as the queue number, where (1) is replaced by another forbidden monochromatic pattern.
- Start the investigation of "union variants" of page numbers, where (2) is replaced by another more restrictive objective.

All three directions require first a study of the existing literature, including a thorough understanding of existing techniques. Afterwards, there are several possible routes to take: Starting with purely theoretical original research with extremal, structural or algorithmic considerations, over the development and implementation of algorithms, to the identification and analysis of potential applications, both within and outside of Graph Drawing.

Requirements.

Students should be familiar with graphs, combinatorial reasoning, and basic computational aspects of theoretical computer science. More importantly, one should be interested in learning classic and recent techniques involving graphs but also willing to dive into mostly unexplored concepts. This endeavor requires curious and creative students that are not afraid to leave the beaten path.