Hyperbolic Unit Disk Graphs

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A *hyperbolic (Euclidean) unit disk graph with threshold radius r* is obtained by placing points into the hyperbolic (Euclidean) plane and connecting each two of them with an edge if their distance is at most r. While the structure of Euclidean unit disk graphs is easy to capture, we do not fully understand the structure of hyperbolic unit disk graphs. In particular, we are interested in the following questions.

Question 1. *Is the treewidth of the neighborhood of every vertex of a hyperbolic unit disk graph bounded by a function of the size of the largest clique?*

Here, the treewidth is a graph parameter that expresses how similar the structure of a graph is to a tree. A positive answer could give rise to algorithmic applications for hyperbolic unit disk graphs with bounded clique number [1].

Closely related, we are interested in so called *product structure*, which describes a graph as a certain graph product of simpler graphs, e.g., planar graphs can be composed from paths, triangles, and a very simple tree-like subclass of planar graphs [2].

Question 2. *Do hyperbolic unit disk graphs with constant clique number admit product structure?*

A positive answer to this question implies a positive answer to the first question, and in addition, would enable us to use the rich and growing literature on product structure theory to improve our understanding of hyperbolic unit disk graphs. On the other side, a negative answer in connection with a positive answer to the first question would contribute to the (still developing) understanding of the notion of product structure.

References

[1] Thomas Bläsius, Tobias Friedrich, Maximilian Katzmann, and Daniel Stephan. "Strongly Hyperbolic Unit Disk Graphs." In: *Symposium on Theoretical Aspects of Computer Science (STACS)*. 2023, 13:1–13:17. DOI: 10.4230/LIPIcs.STACS.2023.13.

[2] Vida Dujmović, Gwenaël Joret, Piotr Micek, Pat Morin, Torsten Ueckerdt, and David R. Wood. "Planar graphs have bounded queue-number." In: *J. ACM* 67.4 (2020), Art. 22, 38. ISSN: 0004-5411. DOI: 10.1145/3385731.