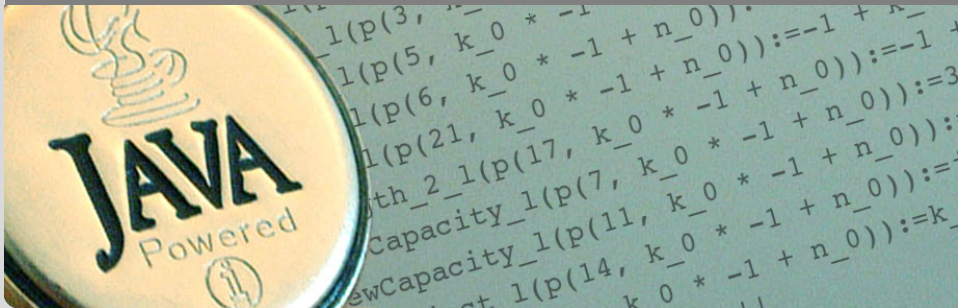


# Applications of Formal Verification

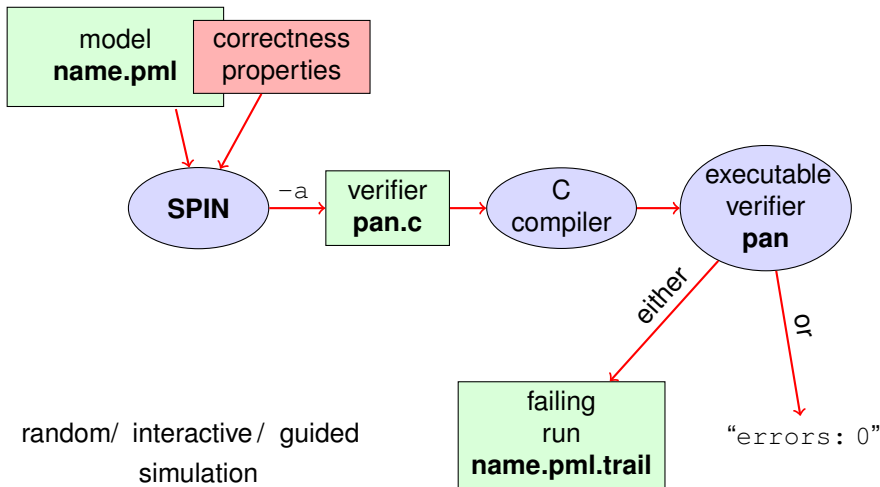
## Model Checking with Temporal Logic

Prof. Dr. Bernhard Beckert · Dr. Vladimir Klebanov | SS 2012

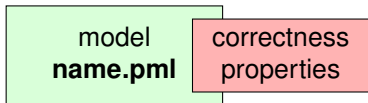
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# Model Checking with SPIN

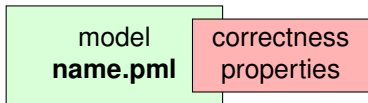


# Stating Correctness Properties



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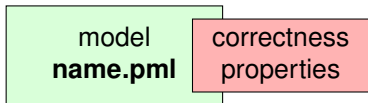


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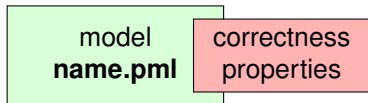
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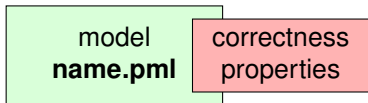
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- never claims
- *temporal logic formulas* (today's main topic)

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model checking of properties formulated in **temporal logic**

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Remark:

in this course, “temporal logic” is synonymous to “*linear temporal logic*” (LTL)

# Beyond Assertions

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- changing assertions is error prone (easily out of synch)
- easy to forget assertions:  
correctness property might be violated at unexpected locations
- many interesting properties not expressible via assertions



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all these are temporal properties  $\Rightarrow$  *use temporal logic*



talking about numerical variables (like in `critical <= 1` or `0 <= i <= len-1`) requires variation of *propositional temporal logic* which we call **Boolean temporal logic**:

- **Boolean expressions** (over PROMELA variables), rather than *propositions*, form basic building blocks of the logic

# Boolean Temporal Logic over PROMELA

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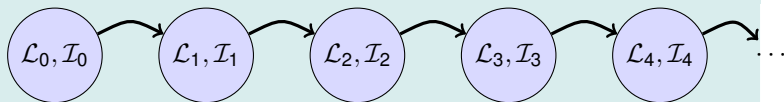
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- if  $\phi$  and  $\psi$  are formulas  $\in For_{BTL}$ , then all of

$$! \phi, \quad \phi \ \&\& \ \psi, \quad \phi \ || \ \psi, \quad \phi \ \rightarrow \ \psi, \quad \phi \ \leftrightarrow \ \psi$$
$$[] \phi, \quad <> \phi, \quad \phi \cup \psi$$

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# Semantics of Boolean Temporal Logic

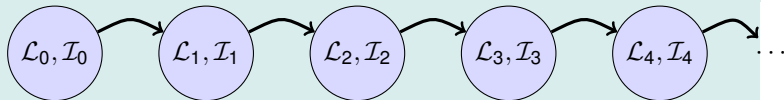
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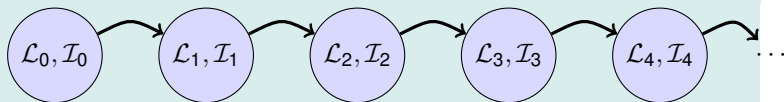


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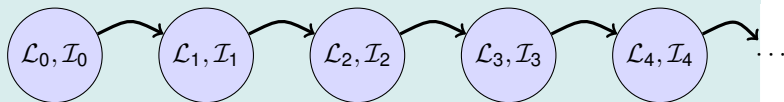
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$\mathcal{L}_j, \mathcal{I}_j \models P @ 1$  iff  $\mathcal{L}_j(P)$  is the location labeled with 1.

Evaluating other formulas  $\in For_{BTL}$  in a run  $\sigma$ : as usual (see the book / “Formale Systeme”).

# Boolean Temporal Logic Support in SPIN

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instead

Boolean expressions must be **abbreviated** using `#define`

What does the following LTL formula mean?

$$[]((Q \ \& \ !R \ \& \ <>R) \ \rightarrow \ (P \ \rightarrow \ (!R \ U \ (S \ \& \ !R))) \ U \ R)$$

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$$[]((Q \ \& \ !R \ \& \ <>R) \ -> (P \ -> (!R \ U \ (S \ \& \ !R))) \ U \ R)$$

P triggers S between Q (e.g., end of system initialization) and R (start of system shutdown).

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example:  $[\ ](\text{critical} \leq 1)$

“it is **guaranteed throughout** each run that at most one process is in its critical section”

or equivalently:

“more than one process being in its critical section will **never happen**”

# Applying Temporal Logic to Critical Section Problem

We want to **verify** `'[] (critical<=1)'` as correctness property of:

```
active proctype P() {  
  do :: /* non-critical activity */  
    atomic {  
      !inCriticalQ;  
      inCriticalP = true  
    }  
    critical++;  
    /* critical activity */  
    critical--;  
    inCriticalP = false  
  od  
}  
  
/* similarly for process Q */
```

# Model Checking a Safety Property with JSPIN

- 1 add `#define mutex (critical <= 1)` to PROMELA file
- 2 open PROMELA file
- 3 enter `[]mutex` in LTL text field
- 4 select `Translate` to create a **'never claim'**, corresponding to the **negation** of the formula
- 5 ensure `Safety` is selected
- 6 select `Verify`
- 7 (if necessary) select `Stop` to terminate too long verification

you may ignore them, but if you are interested:

- a never claim tries to show the user wrong
- it defines, in terms of PROMELA, all **violations** of a wanted correctness property
- it is semantically equivalent to the **negation** of the wanted correctness property
- JSPIN adds the negation for you
- using SPIN directly, you have to add the negation **yourself**

# Model Checking a Safety Property with SPIN directly

## Command Line Execution

*make sure* `#define mutex (critical <= 1)` is in  
`safety1.pml`

```
> spin -a -f "!([] mutex)" safety1.pml  
> gcc -DSAFETY -o pan pan.c  
> ./pan
```



# Temporal MC Without Ghost Variables

We want to **verify mutual exclusion** without using ghost variables

```
#define mutex !(P@cs && Q@cs)

bool inCriticalP = false, inCriticalQ = false;

active proctype P() {
    do :: atomic {
        !inCriticalQ;
        inCriticalP = true
    }
cs:    /* critical activity */
        inCriticalP = false
    od
}
/* similarly for process Q */
/* with same label cs:    */
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Verify '[[]mutex' with JSPIN.
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(with  $csp$  a variable only true in the critical section of  $P$ )

“in each run, process  $P$  visits its critical section **eventually**”

# Applying Temporal Logic to Starvation Problem

We want to **verify** ' $\langle \rangle_{csp}$ ' as correctness property of:

```
active proctype P () {  
  do :: /* non-critical activity */  
    atomic {  
      !inCriticalQ;  
      inCriticalP = true  
    }  
    csp = true;  
    /* critical activity */  
    csp = false;  
    inCriticalP = false  
  od  
}  
  
/* similarly for process Q */  
/* here using csq          */
```

# Model Checking a Liveness Property with JSPIN

- 1 open PROMELA file
- 2 enter `<>csp` in LTL text field
- 3 select `Translate` to create a 'never claim', corresponding to the negation of the formula
- 4 ensure that `Acceptance` is selected  
(SPIN will search for *accepting* cycles through the never claim)
- 5 *for the moment* uncheck `Weak Fairness` (see discussion below)
- 6 select `Verify`



# Verification Fails

Verification fails.

Why?

Verification fails.

Why?

The liveness property on one process 'had no chance'.  
The scheduler **can** unfairly select the other process all the time.

Does the following PROMELA model necessarily terminate?

```
byte n = 0;
bool flag = false;

active proctype P() {
    do :: flag -> break;
      :: else -> n = 5 - n;
    od
}
active proctype Q() {
    flag = true
}
```

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Termination guaranteed only if scheduling is (weakly) fair!

## Definition (Weak Fairness)

A run is called weakly fair iff the following holds:  
each **continuously executable** statement is **executed eventually**.

# Model Checking Liveness with Weak Fairness!

Always switch **Weak Fairness** on when checking for liveness!

- 1 open PROMELA file
- 2 enter `<>csp` in LTL text field
- 3 select `Translate` to create a 'never claim', corresponding to the negation of the formula
- 4 ensure that **Acceptance** is selected  
(SPIN will search for *accepting* cycles through the never claim)
- 5 ensure **Weak Fairness** is checked
- 6 select `Verify`

# Model Checking Liveness with SPIN directly

## Command Line Execution

```
> spin -a -f "!csp" liveness1.pml  
> gcc -o pan pan.c  
> ./pan -a -f
```

# Verification Fails

Verification fails again.

Why?



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Verification fails again.

Why?

Weak fairness is still too weak.

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Note that `!inCriticalQ` is **not** continuously executable!

Verification fails again.

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Note that `!inCriticalQ` is **not** continuously executable!

Designing a fair mutual exclusion algorithm is complicated.

# Literature for this Lecture

Ben-Ari Chapter 5