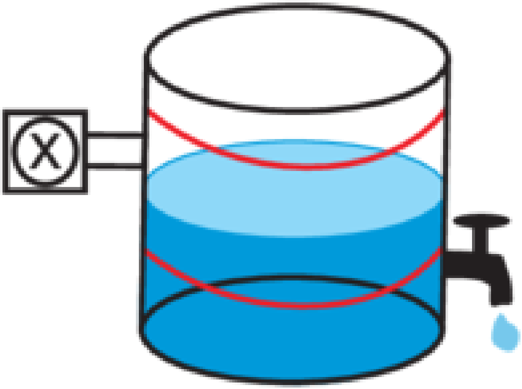
Karlsruhe Institute of Technology

Formale Systeme II: Theorie

SS 2016

Prof. Dr. Bernhard Beckert · Dr. Mattias Ulbrich
Slides by courtesy of André Platzer, CMU

Hybrid automata – Motivation



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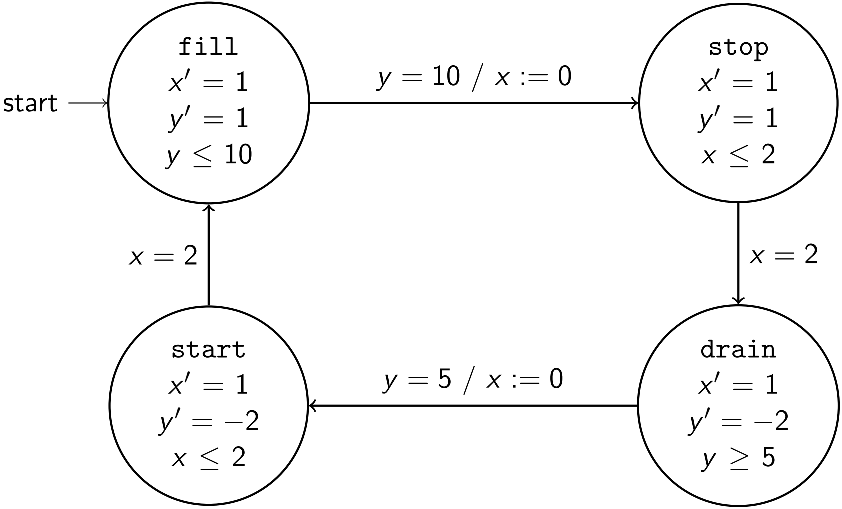
Hybrid automata – Example



Equivalent Hybrid Program

Introducing a state variable q

$$
\begin{array}{llll}
P := & (q = \texttt{fill} \rightarrow & & (x' = 1, y' = 1 \,\&\, y \leq 10 \\
 & & \cup & (?y = 10 \,;\, x := 0 \,;\, q := \texttt{stop}))) \\
\cup & (q = \texttt{stop} \rightarrow & & (x' = 1, y' = 1 \,\&\, x \leq 2 \\
 & & \cup & (?x = 2 \,;\, q := \texttt{drain})) \\
\cup & (q = \texttt{drain} \rightarrow & & (x' = 1, y' = -2 \,\&\, y \geq 5 \\
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\end{array}
$$

Equivalent Hybrid Program

Introducing a state variable q

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\end{array}
$$

Proof obligation

$$\big[x := 0 \,;\, y := 6 \,;\, P^*\big](1 \leq y \wedge y \leq 12)$$

Hybrid Automata

Extension of Finite State Machines (*Henzinger*, 1990s)

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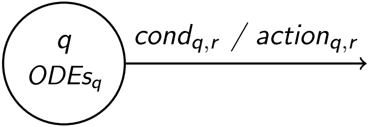
State $q \in S$ with edge to $r \in S$:



ODEs may have domain constraints

Hybrid Automata

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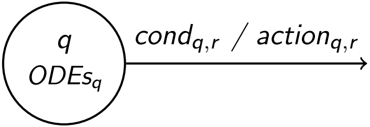

ODEs may have domain constraints

Semantics (Idea)

1. Sequence of *edge steps* and *time steps (flow)*

Hybrid Automata

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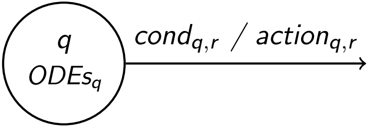

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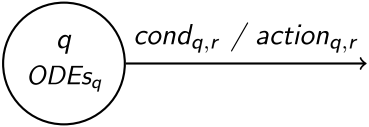

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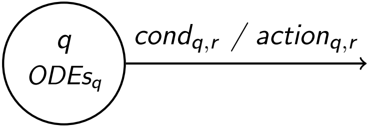

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2. during flow: variables evolve according to $ODEs_q$
3. discrete state changes at t_i from q_i to q_{i+1}:
   $cond_{q_i}$ must hold, $action_{q_i,q_{i+1}}$ is performed
4. edge: condition $cond_{q,r}$ satisfied, $action_{q,r}$ performed discretely, new state is r

Rectangular Hybrid Automata

Rectangular condition

A rectangular condition on *Var* is a conjunction of atoms of the form $x \leq const$ or $x \geq const$ for variables $x \in Var$.

Rectangular automata

A hybr. automaton is called *rectangular* if

Rectangular Hybrid Automata

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- every *action* is a sequence of assignments $x := const$
- every *ODE* is a rectangular condition on the derivatives $x', \ldots$
- every domain constraint is a rectangular condition

Rectangular Hybrid Automata

Decidability

The safety problem for rectangular automata w.r.t. to rectangular safety invariants is decidable (in PSPACE).

["What's Decidable about Hybrid Automata?", Henzinger et al. 1998]

Proof by reduction to *timed automata* $\rightarrow$ lecture FS2: Application

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Proof by reduction to *timed automata* $\rightarrow$ lecture FS2: Application

Undecidablity result

The safety problem is undecidable for hybrid automata with general linear ODEs.

Caution with Specification

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Validity

A hybrid model M for a cyberphysical system S is *valid* w.r.t. a property P if $M \models P$ implies $S \models P$.

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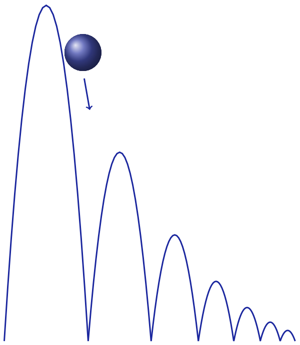
A hybrid model M for a cyberphysical system S is *valid* w.r.t. a property P if $M \models P$ implies $S \models P$.

Validity of hybrid system is important

It *always* includes an abstraction of reality.
Often this abstraction has to be crafted towards the problem.

How Quantum Met Achilles and His Tortoise



Example (Quantum the Bouncing Ball)

$$\big(x' = v, v' = -g \,\&\, x \geq 0;$$
$$\mathtt{if}(x = 0)\, v := -cv\big)^*$$

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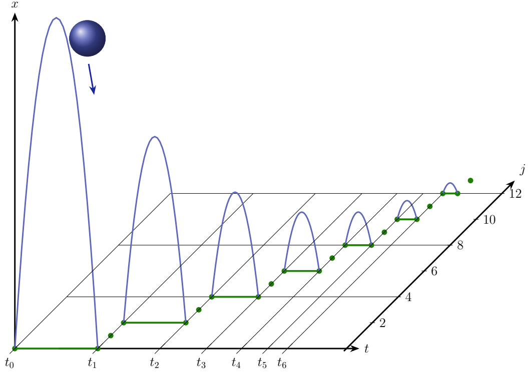

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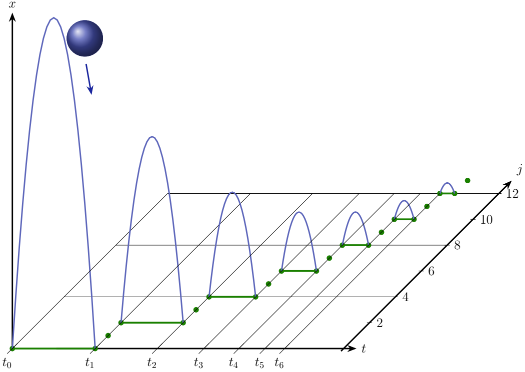

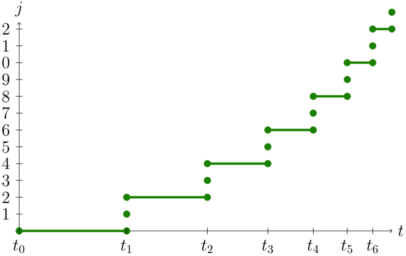

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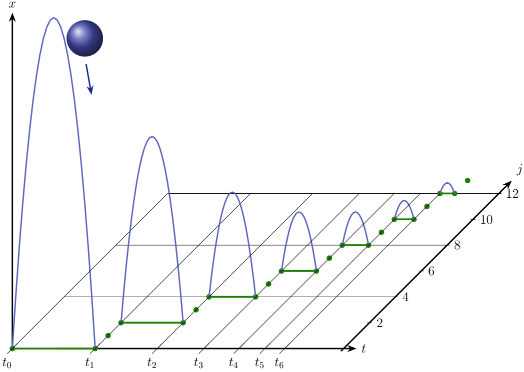
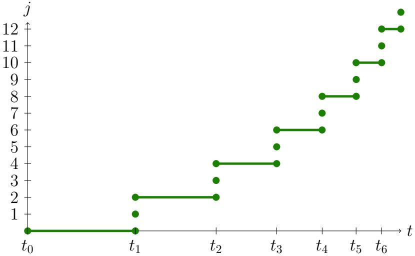
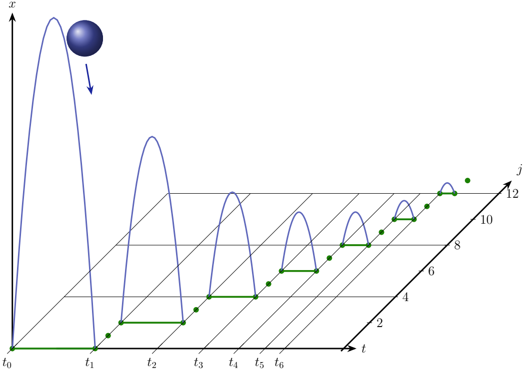


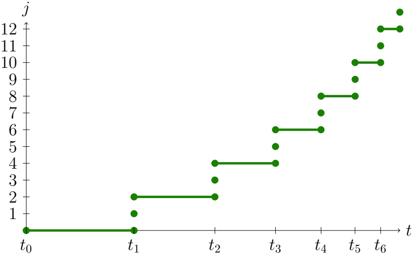


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

How Quantum Met Achilles and His Tortoise





Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \sum_{i=0}^{\infty} \frac{1}{2^i}$$

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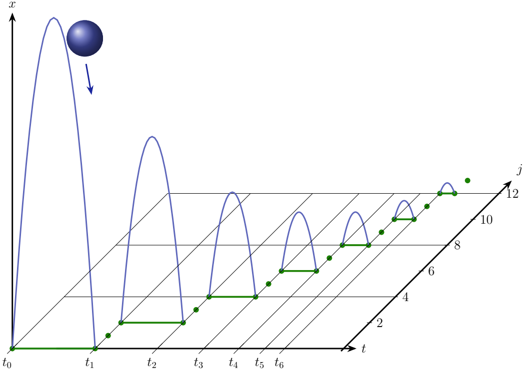

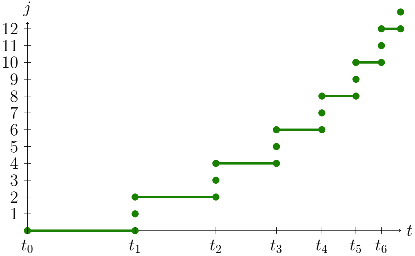

Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}}$$

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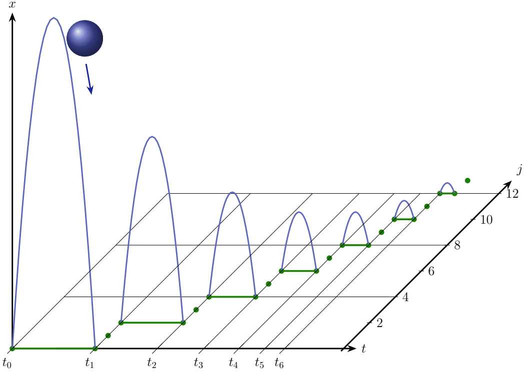

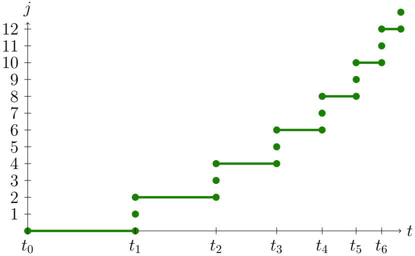

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$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2$$

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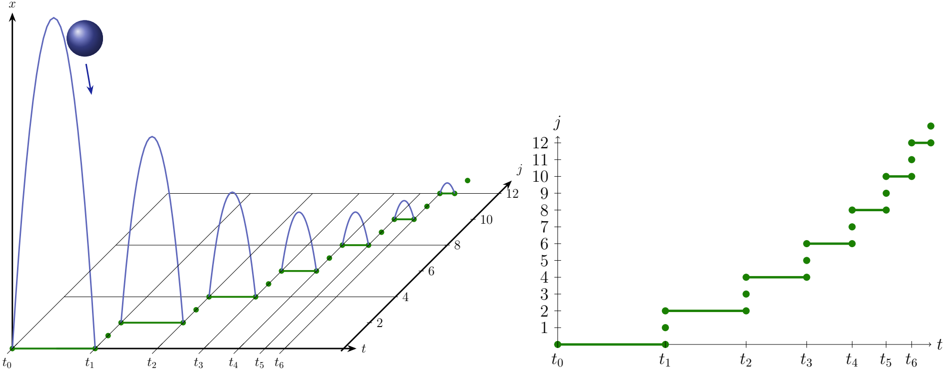

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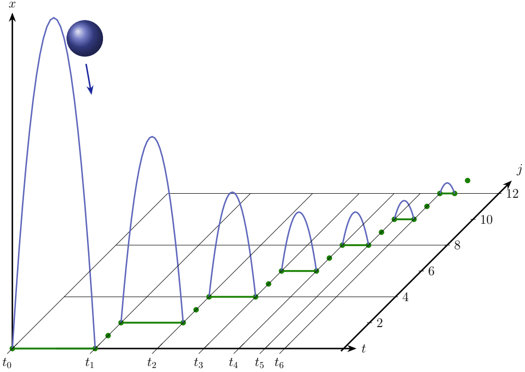

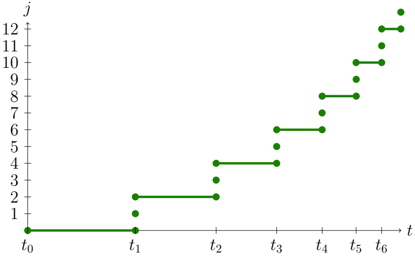

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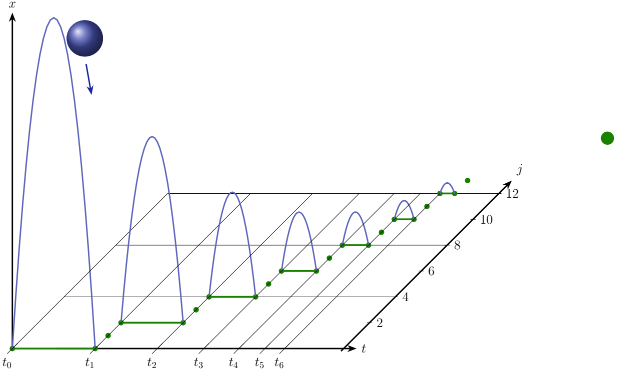

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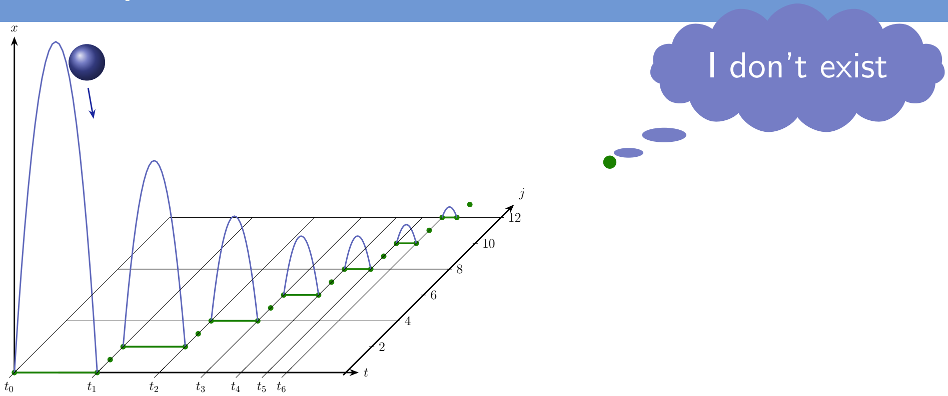

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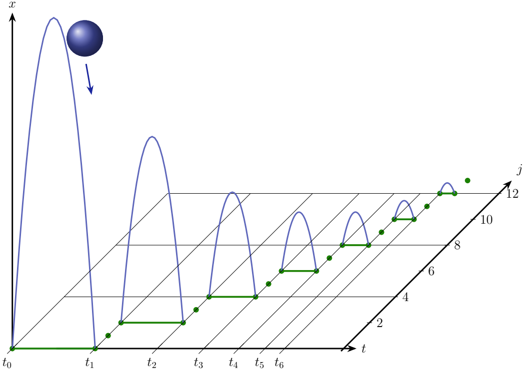

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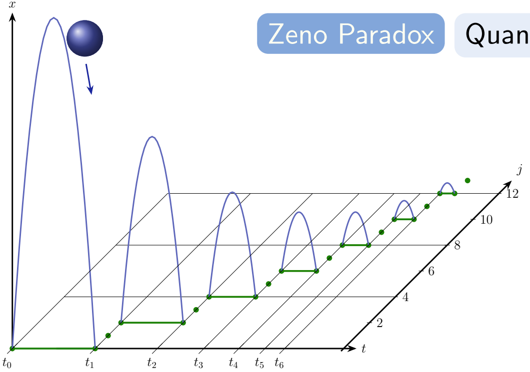
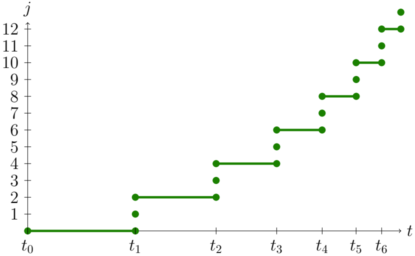
How Quantum Met Achilles and His Tortoise

Zeno Paradox

Quantum's model causes a time freeze





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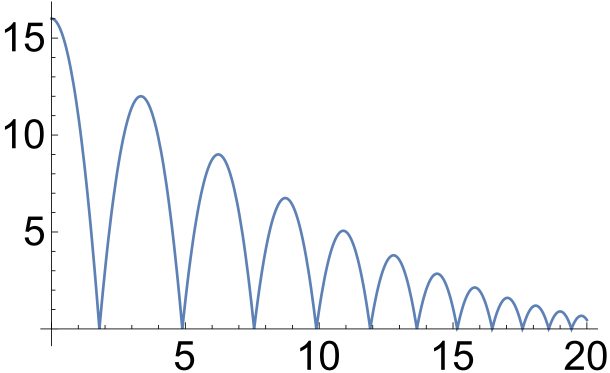
Quantum the Safely Bored Bouncing Ball

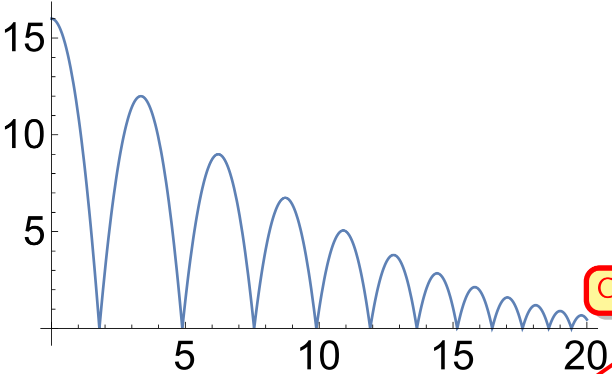


Proposition (Quantum can bounce around safely)

$$0 \le x \land x = H \land v = 0 \land g > 0 \land 1 \ge c \ge 0 \to$$
$$[(x' = v, v' = -g \,\&\, x \ge 0; (?x{=}0; v := -cv \cup ?x{\ne}0))^*](0 \le x \land x \le H)$$

Proof @invariant$(2gx = 2gH - v^2 \land x \ge 0)$

Quantum the Safely Bored Bouncing Ball



Can be improved...

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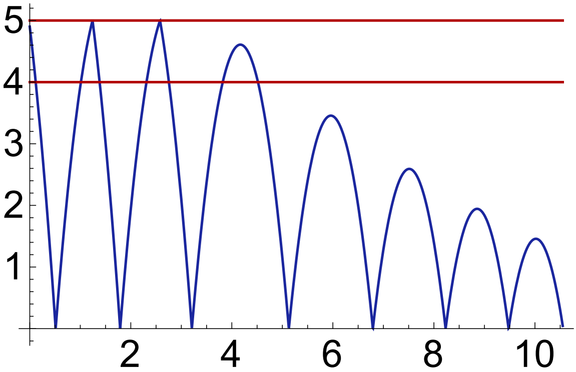
Quantum the Daring Ping Pong Ball



Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\Big[\big(x' = v, v' = -g \,\&\, x \ge 0;$$
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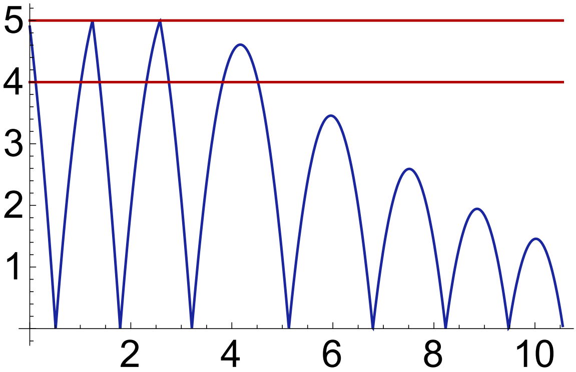
Quantum the Daring Ping Pong Ball



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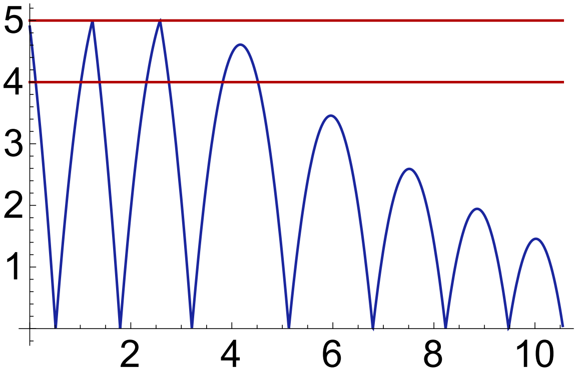
Proof? Ask René Descartes

Cartesian Doubt: René Descartes's Cartesian Demon 1641

Outwit the Cartesian Demon

Skeptical about the truth of all beliefs until justification has been found.

Quantum the Daring Ping Pong Ball



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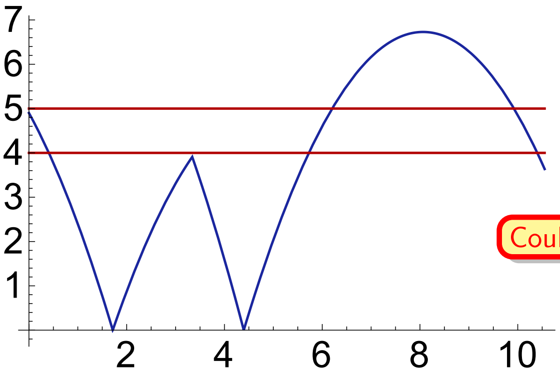
Quantum the Daring Ping Pong Ball



Could run instead of control

Conjecture (Quantum can play ping pong safely)

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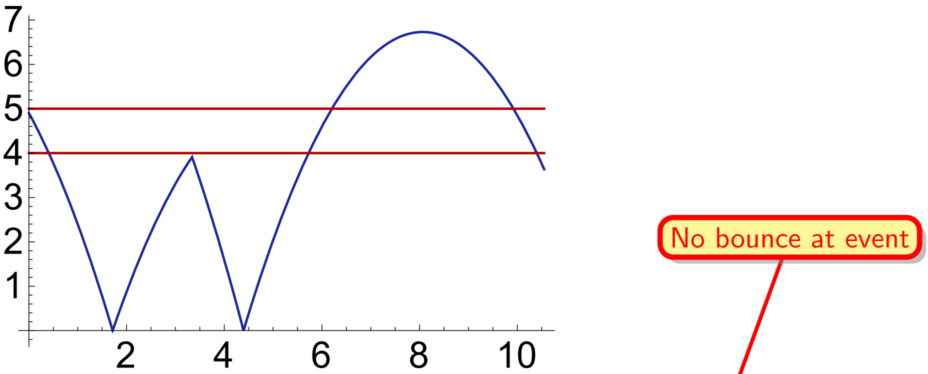
Proof? Ask René Descartes who says no!

Quantum the Daring Ping Pong Ball



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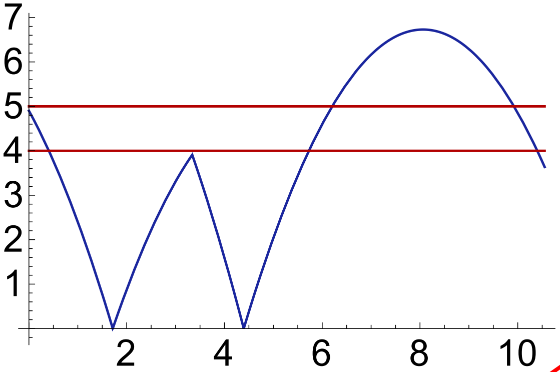
Quantum the Daring Ping Pong Ball



Could miss this event

Conjecture (Quantum can play ping pong safely)

$$0 \le x \wedge x \le 5 \wedge v \le 0 \wedge g > 0 \wedge 1 \ge c \ge 0 \wedge f \ge 0 \rightarrow$$
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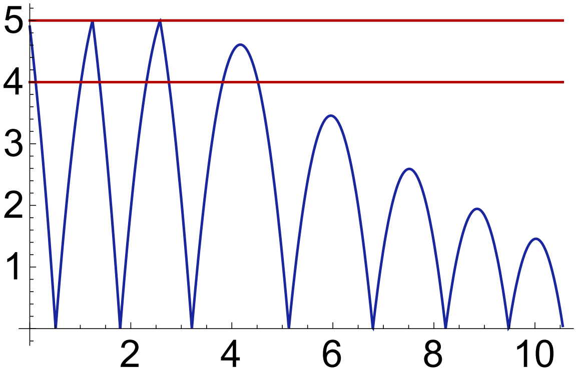
Proof? Ask René Descartes who says no!

Quantum the Deterministically Daring Ping Pong Ball



Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
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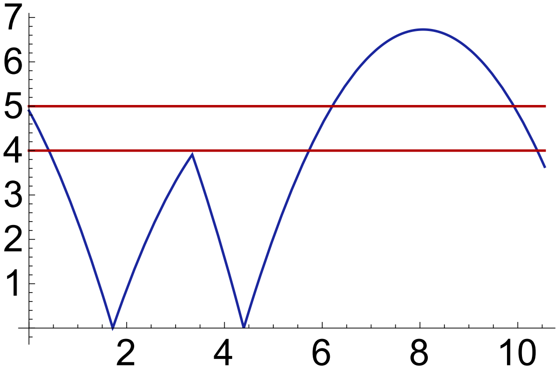

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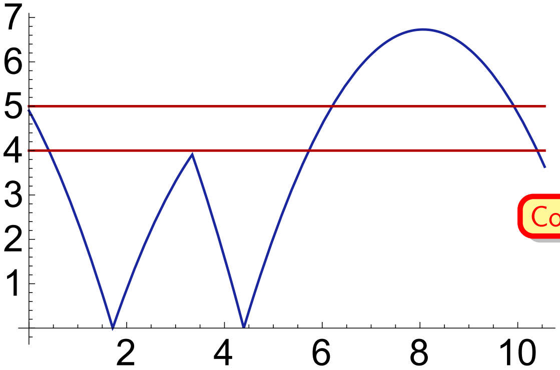

Could also miss if-then event

Conjecture (Quantum can play ping pong safely)

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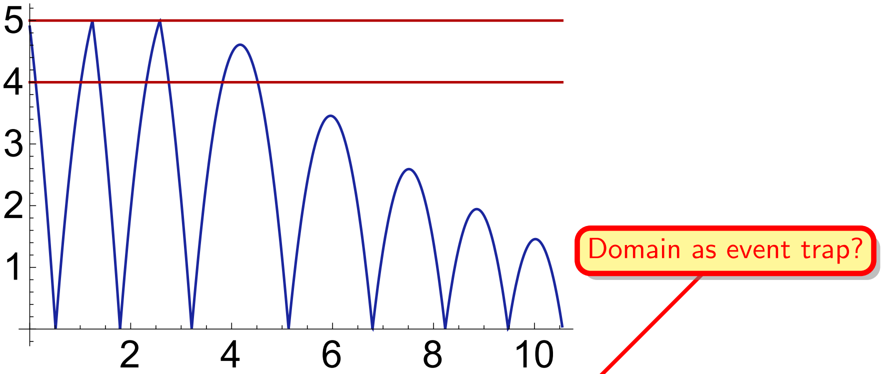

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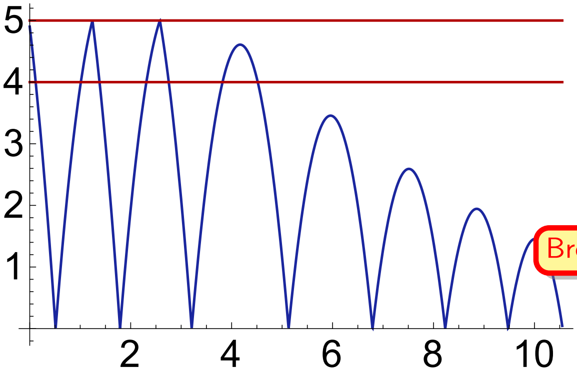

Broken physics: Always event

Conjecture (Quantum can play ping pong safely)

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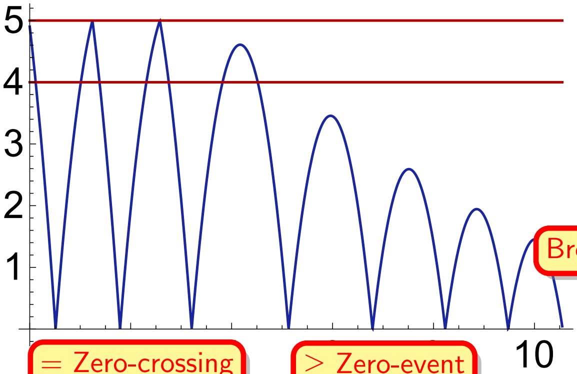

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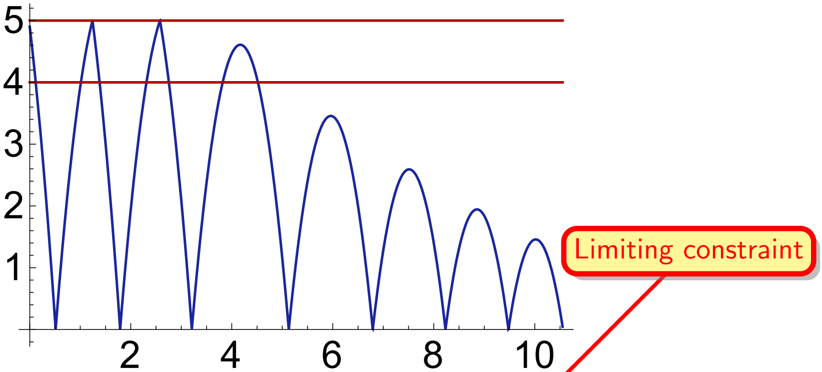

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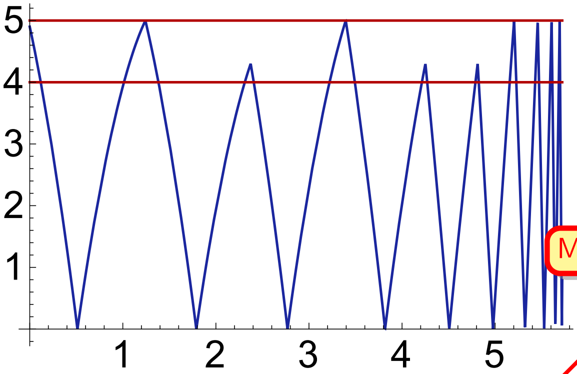

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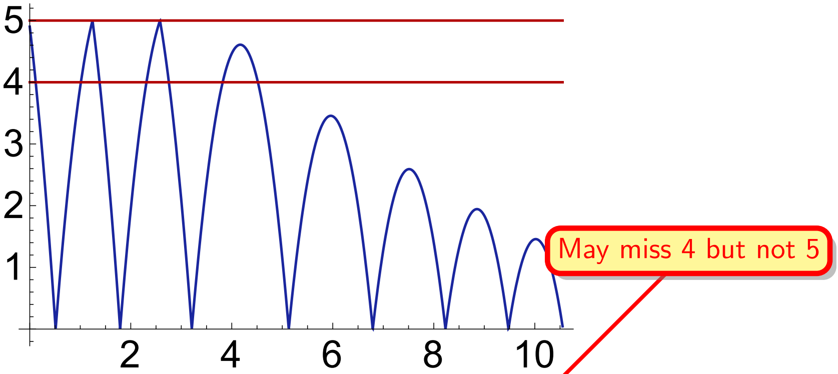

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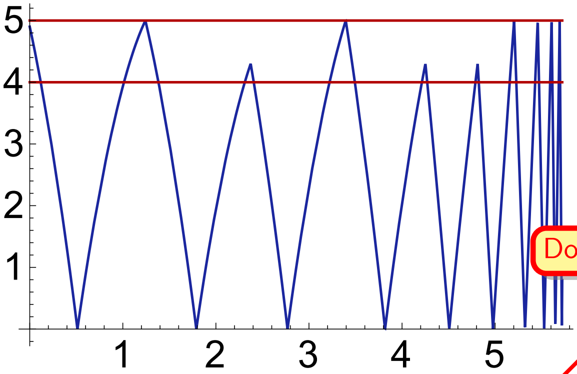

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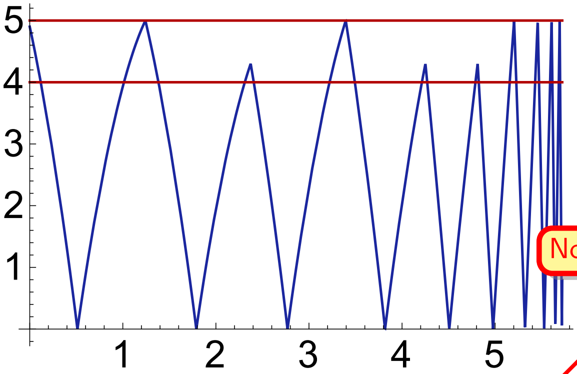
Quantum the Deterministically Daring Ping Pong Ball



Non-negotiable physics

Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\big[\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5\};$$
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Proof? Ask René Descartes who says yes! But meant to say no!

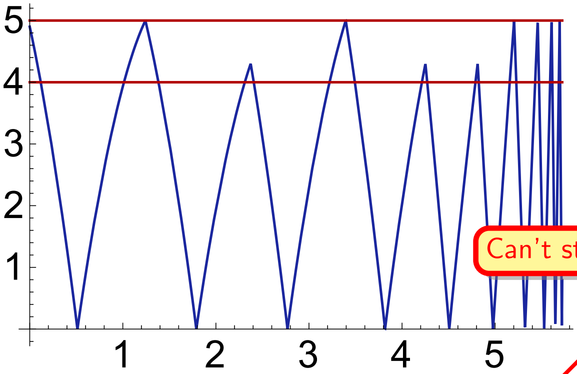
On the Nuisance of Nuances of Physics

Non-negotiability of Physics

1. Making systems safe by construction is a great idea. For control!
2. Not by changing the laws of physics around.
3. Physics is unpleasantly non-negotiable.
4. If models are safe because we forgot to include all behavior of physical reality, then correctness statements only hold in that other universe.

Despite control — We don't get to boss physics around

We don't make this world any safer by writing CPS programs for another universe.



Can't stop the world for an event

Conjecture (Quantum can play ping pong safely)

$$0 \leq x \land x \leq 5 \land v \leq 0 \land g > 0 \land 1 \geq c \geq 0 \land f \geq 0 \rightarrow$$
$$\big[\big(\{x' = v, v' = -g \,\&\, x \geq 0 \land x \leq 5\};$$
$$\texttt{if}(x{=}0)\, v := -cv \,\texttt{else}\,\texttt{if}(4{\leq}x{\leq}5)\, v := -fv\big)^*\big](0{\leq}x{\leq}5)$$

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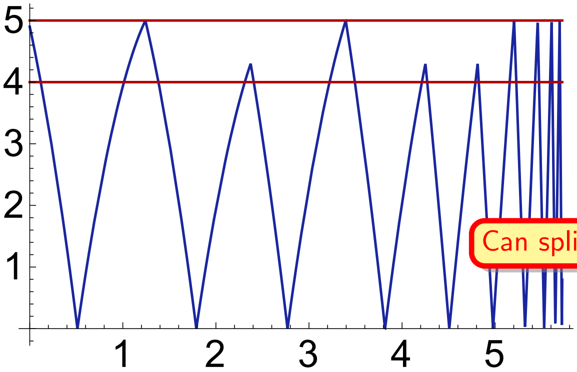
Quantum the Deterministically Daring Ping Pong Ball



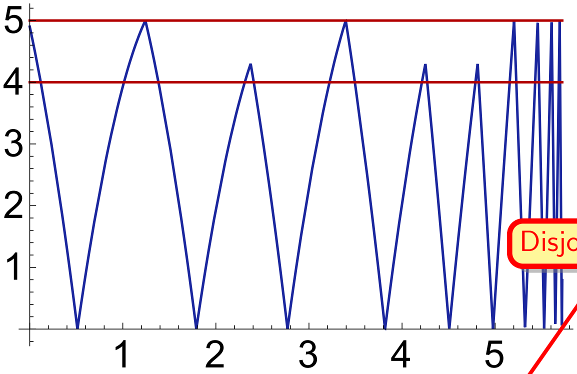
Can split the world for an event

Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\big[\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x{\le}5 \;\cup\; x' = v, v' = -g \,\&\, x{>}5\};$$
$$\texttt{if}(x{=}0)\, v := -cv \;\texttt{else}\;\texttt{if}(4{\le}x{\le}5)\, v := -fv\big)^*\big](0{\le}x{\le}5)$$

Proof? Ask René Descartes

Quantum the Deterministically Daring Ping Pong Ball



Disjoint domains

Shattered the world

Conjecture (Quantum can play ping pong safely)

$$0 \le x \wedge x \le 5 \wedge v \le 0 \wedge g > 0 \wedge 1 \ge c \ge 0 \wedge f \ge 0 \rightarrow$$
$$\big[\big(\{x' = v, v' = -g \,\&\, x \ge 0 \wedge x \le 5 \;\cup\; x' = v, v' = -g \,\&\, x > 5\};$$
$$\mathtt{if}(x=0)\, v := -cv \;\mathtt{else}\;\mathtt{if}(4 \le x \le 5)\, v := -fv\big)^*\big](0 \le x \le 5)$$

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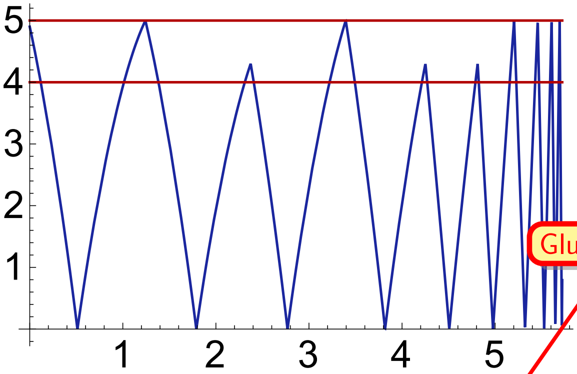

Glue domains

Reunite the world

Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \rightarrow$$
$$\Big[\Big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5 \;\cup\; x' = v, v' = -g \,\&\, x \ge 5\};$$
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Proof? Ask René Descartes

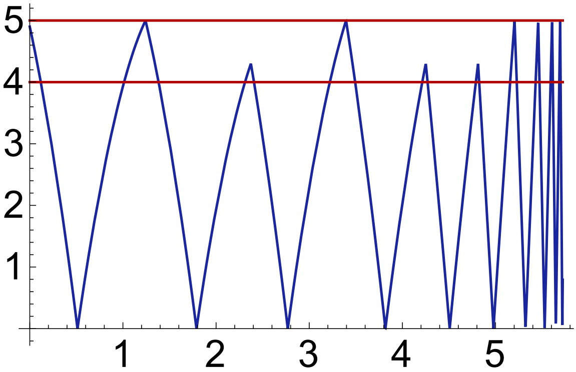
Connected Evolution Domains

Connected evolution domains

1. Evolution domain constraints need care.
2. Determine regions within which the system can evolve.
3. Disconnected/disjoint disallows continuous transitions.

1. Splitting the state space into different regions to detect events is fine.
2. Destroying the world is not.
3. Not even by poking infinitesimal holes into the time-space continuum.

Quantum the Deterministically Daring Ping Pong Ball



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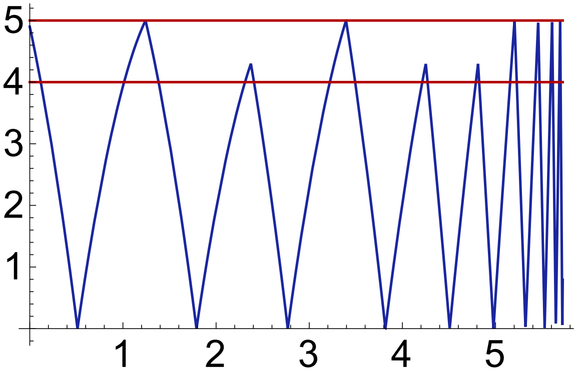

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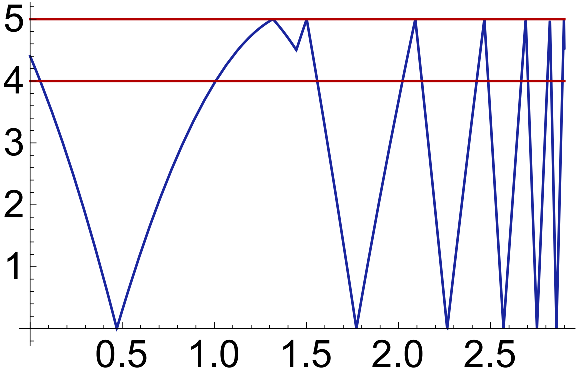

Multi-fire

Conjecture (Quantum can play ping pong safely)

$$0 \le x \land x \le 5 \land v \le 0 \land g > 0 \land 1 \ge c \ge 0 \land f \ge 0 \to$$
$$\big[\big(\{x' = v, v' = -g \,\&\, x \ge 0 \land x \le 5 \;\cup\; x' = v, v' = -g \,\&\, x \ge 5\};$$
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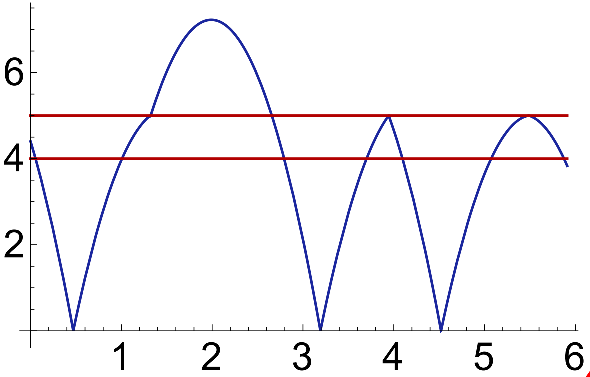

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Proof? Ask René Descartes who definitely says no!

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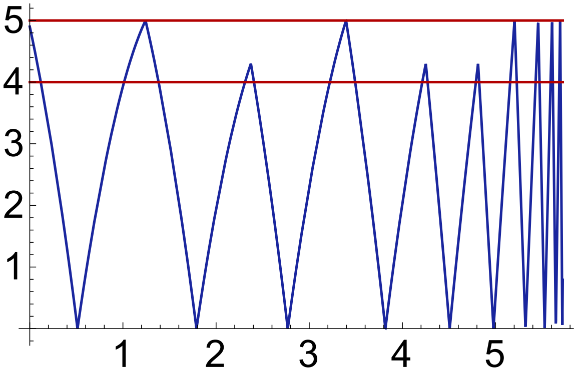

Only upsense event

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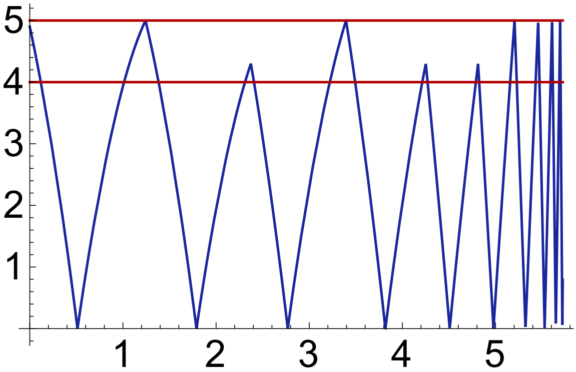
Proof? Ask René Descartes

Multi-firing Events

Multi-firing of events

1. If the same event is detected multiple times:
2. Are multiple responses acceptable?
3. Or is a single response crucial?



control: robust, all cases

physics: precise

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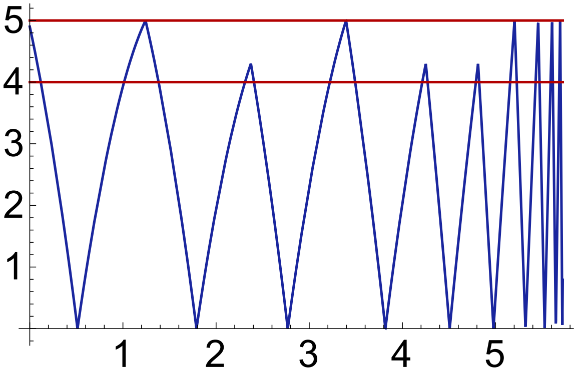
Physics vs. Control: Classification



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Hybrid Model Composition

Separate concerns in hybrid programs

Event-triggered

$$(Control\ ;\ ODEs\ ;\ discrPhysics)^*$$

Time-triggered

For a time slice δ

$$(Control\ ;$$
$$t := 0\ ;\ (t < \delta?\ ;\ ODEs, t' = 1\ \&\ t \leq \delta\ ;\ discrPhysics)^*)^*$$