Fair Allocation of Goods

Consider a set of agents and a set of goods. Each agent has her own preferences regarding the allocation of goods to agents. Examples:

- allocation of resources amongst members of our society
- allocation of bandwith to processes in a communication network
- allocation of compute time to scientists on a super-computer
- ...

We will focus on one specific model studied in the literature, with a single good that can be divided into arbitrarily small pieces ...

Cake Cutting

A classical example for a problem of collective decision making:

> *We have to divide a cake with different toppings amongst n agents by means of parallel cuts. Agents have different preferences regarding the toppings (additive utility functions).*

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The exact details of the formal model are not important for this short exposition. You can look them up in my lecture notes (cited below).

U. Endriss. *Lecture Notes on Fair Division*. Institute for Logic, Language and Computation, University of Amsterdam, 2009/2010.

Cut-and-Choose

The classical approach for dividing a cake between *two agents:*

- *One agent cuts the cake in two pieces (she considers to be of equal value), and the other chooses one of them (the piece she prefers).*

The cut-and-choose protocol is *fair* in the sense of guaranteeing a property known as *proportionality:*

- Each agent is *guaranteed* at least one half (general: $1/n$), according to her own valuation.
- Discussion: In fact, the first agent (if she is risk-averse) will receive exactly $1/2$, while the second will usually get more.

What if there are *more than two* agents?

The Banach-Knaster Last-Diminisher Protocol

In the first ever paper on fair division, Steinhaus (1948) reports on a *proportional* protocol for n *agents* due to Banach and Knaster.

(1) Agent 1 cuts off a piece (that she considers to represent $1/n$).

(2) That piece is passed around the agents. Each agent either lets it pass (if she considers it too small) or trims it down further (to what she considers $1/n$).

(3) After the piece has made the full round, the last agent to cut something off (the "last diminisher") is obliged to take it.

(4) The rest (including the trimmings) is then divided amongst the remaining $n-1$ agents. Play cut-and-choose once $n = 2$. ✓

Each agent is guaranteed a *proportional* piece. Requires $O(n^2)$ cuts. May not be *contiguous* (unless you always trim "from the right").

H. Steinhaus. The Problem of Fair Division. *Econometrica*, 16:101–104, 1948.