

Fair Allocation of Goods

Consider a set of agents and a set of goods. Each agent has her own preferences regarding the allocation of goods to agents. Examples:

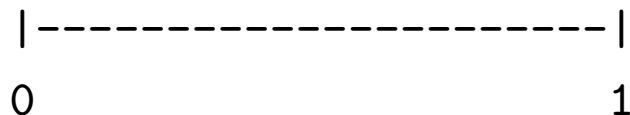
- allocation of resources amongst members of our society
- allocation of bandwidth to processes in a communication network
- allocation of compute time to scientists on a super-computer
- ...

We will focus on one specific model studied in the literature, with a single good that can be divided into arbitrarily small pieces ...

Cake Cutting

A classical example for a problem of collective decision making:

*We have to divide a **cake** with different toppings amongst n **agents** by means of parallel cuts. Agents have different preferences regarding the toppings (**additive utility functions**).*



The exact details of the formal model are not important for this short exposition. You can look them up in my lecture notes (cited below).

U. Endriss. *Lecture Notes on Fair Division*. Institute for Logic, Language and Computation, University of Amsterdam, 2009/2010.

Cut-and-Choose

The classical approach for dividing a cake between *two agents*:

- ▶ One agent *cuts* the cake in two pieces (she considers to be of equal value), and the other *chooses* one of them (the piece she prefers).

The cut-and-choose protocol is *fair* in the sense of guaranteeing a property known as *proportionality*:

- Each agent is *guaranteed* at least one half (general: $1/n$), according to her own valuation.
- Discussion: In fact, the first agent (if she is risk-averse) will receive exactly $1/2$, while the second will usually get more.

What if there are *more than two* agents?

The Banach-Knaster Last-Diminisher Protocol

In the first ever paper on fair division, Steinhaus (1948) reports on a *proportional* protocol for n agents due to Banach and Knaster.

- (1) Agent 1 cuts off a piece (that she considers to represent $1/n$).
- (2) That piece is passed around the agents. Each agent either lets it pass (if she considers it too small) or trims it down further (to what she considers $1/n$).
- (3) After the piece has made the full round, the last agent to cut something off (the “last diminisher”) is obliged to take it.
- (4) The rest (including the trimmings) is then divided amongst the remaining $n-1$ agents. Play cut-and-choose once $n = 2$. ✓

Each agent is guaranteed a *proportional* piece. Requires $O(n^2)$ cuts. May not be *contiguous* (unless you always trim “from the right”).

H. Steinhaus. The Problem of Fair Division. *Econometrica*, 16:101–104, 1948.