

Matching

In a variant of the fair allocation problem, we try to match each agent with a single item—which may have preferences itself. Examples:

- children to schools
- junior doctors to hospitals
- kidney patients to kidney donors
- ...

We now briefly look into *the* classical matching problem.

The Stable Marriage Problem

We are given:

- n *men* and n *women*
- each has a linear *preference* ordering over the opposite sex

We seek:

- a *stable* matching of men to women: no man and woman should want to divorce their assigned partners and run off with each other

The Gale-Shapley Algorithm

Theorem 1 (Gale and Shapley, 1962) *There exists a stable matching for any combination of preferences of men and women.*

The *Gale-Shapley “deferred acceptance” algorithm* for computing a stable matching works as follows:

- In each round, each man who is not yet engaged proposes to his favourite amongst the women he has not yet proposed to.
- In each round, each woman picks her favourite from the proposals she’s receiving and the man she’s currently engaged to (if any).
- Stop when everyone is engaged.

D. Gale and L.S. Shapley. College Admissions and the Stability of Marriage. *American Mathematical Monthly*, 69:9–15, 1962.