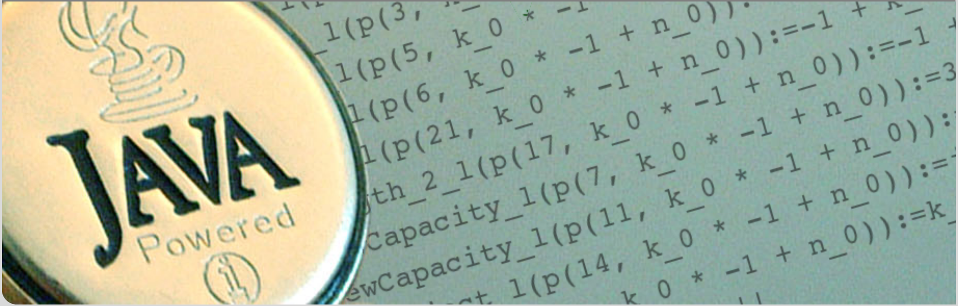
Karlsruhe Institute of Technology

Applications of Formal Verification

Functional Verification of Java Programs: Java Dynamic Logic

Bernhard Beckert · Mattias Ulbrich | SS 2019

KIT – INSTITUT FÜR THEORETISCHE INFORMATIK



1. JAVA CARD DL
2. Sequent Calculus
3. Rules for Programs: Symbolic Execution
4. A Calculus for 100% JAVA CARD
5. Loop Invariants

Syntax and Semantics

Syntax

- Basis: Typed first-order predicate logic
- Modal operators $\langle p \rangle$ and $[p]$ for each (JAVA CARD) program p
- Class definitions in background (not shown in formulas)

Semantics (Kripke)

Modal operators allow referring to the final state of p:

- $[p]F$: If p terminates normally, then F holds in the final state ("partial correctness")
- $\langle p \rangle F$: p terminates normally, and F holds in the final state ("total correctness")

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Why Dynamic Logic?

- Transparency wrt target programming language
- Encompasses Hoare Logic
- More expressive and flexible than Hoare logic
- Symbolic execution is a natural interactive proof paradigm

- Programs are “first-class citizens”
- Real Java syntax

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Hoare triple $\{\psi\}\ \alpha\ \{\phi\}$ equiv. to DL formula $\psi \rightarrow [\alpha]\phi$

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Not merely partial/total correctness:

- can employ programs for specification (e.g., verifying program transformations)
- can express security properties (two runs are indistinguishable)
- extension-friendly (e.g., temporal modalities)

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Dynamic Logic Example Formulas

$(\texttt{balance} >= c \land \texttt{amount} > 0) \rightarrow$
$\langle \texttt{charge(amount);} \rangle\, \texttt{balance} > c$

$\langle \texttt{x = 1;} \rangle ([\texttt{while (true) \{\}}]\mathit{false})$

- Program formulas can appear nested

$\texttt{\textbackslash forall}\ \mathit{int\ val}; ((\langle \texttt{p} \rangle \texttt{x} = \mathit{val}) \longleftrightarrow (\langle \texttt{q} \rangle \texttt{x} = \mathit{val}))$

- p, q equivalent relative to computation state restricted to x

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Dynamic Logic Example Formulas

```
  a != null
->
  <
    int max = 0;
    if ( a.length > 0 ) max = a[0];
    int i = 1;
    while ( i < a.length ) {
      if ( a[i] > max ) max = a[i];
      ++i;
    }
  >(
      \forall int j; (j >= 0 & j < a.length -> max >= a[j])
      &
      (a.length > 0 ->
        \exists int j; (j >= 0 & j < a.length & max = a[j]))
    )
```

Variables

- Logical variables disjoint from program variables
  - No quantification over program variables
  - Programs do not contain logical variables
  - “Program variables” actually non-rigid functions

Validity

A JAVA CARD DL formula is valid iff it is true in all states.

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Sequents and their Semantics

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$$\underbrace{\psi_1, \ldots, \psi_m}_{\textit{Antecedent}} \quad \Longrightarrow \quad \underbrace{\phi_1, \ldots, \phi_n}_{\textit{Succedent}}$$

where the ϕ_i, ψ_i are formulae (without free variables)

Semantics

Same as the formula

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Sequent Rules

General form

$$\text{RULE\_NAME}\ \frac{\overbrace{\Gamma_1 \Longrightarrow \Delta_1 \quad \cdots \quad \Gamma_r \Longrightarrow \Delta_r}^{\text{Premisses}}}{\underbrace{\Gamma \Longrightarrow \Delta}_{\text{Conclusion}}}$$

($r = 0$ possible: closing rules)

Soundness

If all premisses are valid, then the conclusion is valid

Use in practice

Goal is matched to conclusion

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NOT_LEFT $$\frac{\Gamma \Longrightarrow A, \Delta}{\Gamma, \neg A \Longrightarrow \Delta}$$

IMP_LEFT $$\frac{\Gamma \Longrightarrow A, \Delta \qquad \Gamma, B \Longrightarrow \Delta}{\Gamma, A \to B \Longrightarrow \Delta}$$

CLOSE_GOAL $$\frac{}{\Gamma, A \Longrightarrow A, \Delta}$$

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ALL_LEFT $$\frac{\Gamma, \texttt{\textbackslash forall}\ t\ x; \phi,\ \{x/e\}\phi \Longrightarrow \Delta}{\Gamma, \texttt{\textbackslash forall}\ t\ x; \phi \Longrightarrow \Delta}$$

where e var-free term of type $t' \prec t$

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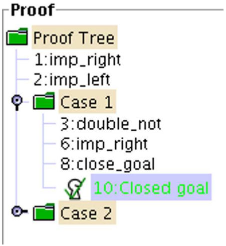
Sequent Calculus Proofs

Proof tree

- Proof is tree structure with goal sequent as root
- Rules are applied from conclusion (old goal) to premisses (new goals)
- Rule with no premiss closes proof branch
- Proof is finished when all goals are closed



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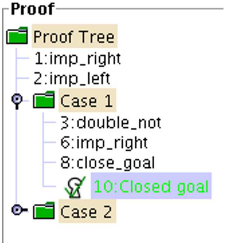

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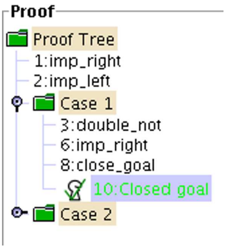

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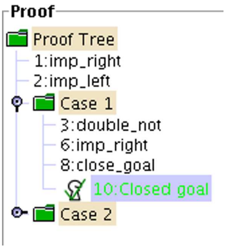

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- Sequent rules for program formulas?
- What corresponds to top-level connective in a program?

The Active Statement in a Program

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Rules for Symbolic Program Execution

If-then-else rule

$$\frac{\Gamma, B = \mathit{true} \Longrightarrow \langle p \;\; \omega\rangle\phi, \Delta \qquad \Gamma, B = \mathit{false} \Longrightarrow \langle q \;\; \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \texttt{if (}B\texttt{) \{ }p\texttt{ \} else \{ }q\texttt{ \}}\;\; \omega\rangle\phi, \Delta}$$

Complicated statements/expressions are simplified first, e.g.

$$\frac{\Gamma \Longrightarrow \langle \texttt{v=y; y=y+1; x=v;} \;\; \omega\rangle\phi, \Delta}{\Gamma \Longrightarrow \langle \texttt{x=y++;} \;\; \omega\rangle\phi, \Delta}$$

Simple assignment rule

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Treating Assignment with "Updates"

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Updates

syntactic elements in the logic – (explicit substitutions)

Elementary Updates

$$\{loc := val\}\,\phi$$

where

- *loc* is a program variable
- *val* is an expression type-compatible with *loc*

Parallel Updates

$$\{loc_1 := t_1 \parallel \cdots \parallel loc_n := t_n\}\,\phi$$

no dependency between the *n* components (but 'last wins' semantics)

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Updates are

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Symbolic Execution with Updates
(by Example)

$$x < y \Longrightarrow x < y$$

$$\vdots$$

$$x < y \Longrightarrow \{x:=y \,\|\, y:=x\}\langle\rangle\, y < x$$

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$$\texttt{x} < \texttt{y} \Longrightarrow \{\texttt{t:=x} \parallel \texttt{x:=y} \parallel \texttt{y:=x}\}\langle\rangle\, \texttt{y} < \texttt{x}$$

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$$\vdots$$

$$\Longrightarrow \texttt{x} < \texttt{y} \to \langle\texttt{int t=x; x=y; y=t;}\rangle\, \texttt{y} < \texttt{x}$$

Symbolic Execution with Updates
(by Example)

$$\mathtt{x} < \mathtt{y} \Longrightarrow \mathtt{x} < \mathtt{y}$$

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The theory of arrays

An abstract datatype $\mathit{Array}(\mathbb{I}, \mathbb{V})$

Types: Indices $\mathbb{I}$, Values $\mathbb{V}$

Function symbols:

- $\mathit{select} : \mathit{Array}(\mathbb{I}, \mathbb{V}) \times \mathbb{I} \to \mathbb{V}$
- $\mathit{store} : \mathit{Array}(\mathbb{I}, \mathbb{V}) \times \mathbb{I} \times \mathbb{V} \to \mathit{Array}(\mathbb{I}, \mathbb{V})$

Axioms

$$\forall a, i, v. \quad \mathit{select}(\mathit{store}(a, i, v), i) = v$$

$$\forall a, i, j, v. \; i \neq j \to \mathit{select}(\mathit{store}(a, i, v), j) = \mathit{select}(a, j)$$

Intuition

$\mathcal{D}(\mathit{Array}(\mathbb{I}, \mathbb{V}))$ represents the set of functions $\mathcal{D}(\mathbb{I}) \to \mathcal{D}(\mathbb{V})$

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The theory of arrays

An abstract data

Types: Indices $\mathbb{I}$,

Function symbol

- *select* : *Array*
- *store* : *Array*(

Photo by "null0" (www.flickr.com/photos/null0/272015955)

John McCarthy (1927–2011):
Theory of arrays is decidable

Axioms

$\forall a, i, v.$

$$\forall a, i, j, v.\ i \neq j \rightarrow select(store(a, i, v), j) = select(a, j)$$

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Program State Representation

Local program variables

Modeled as non-rigid constants

Heap

Modeled with theory of arrays: $\mathbb{I} = Object \times Field$, $\mathbb{V} = Any$

$heap$:	$Heap$ (the heap in the current state)
$select$:	$Heap \times Object \times Field \to Any$
$store$:	$Heap \times Object \times Field \times Any \to Heap$

Some special program variables

`self`	the current receiver object (`this` in Java)
`exc`	the currently active exception (`null` if none thrown)
`result`	the result of the method invocation

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Supported Java Features

- method invocation with polymorphism/dynamic binding
- object creation and initialisation
- arrays
- abrupt termination
- throwing of NullPointerExceptions, etc.
- bounded integer data types
- transactions

All JAVA CARD language features are fully addressed in KeY

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Java—A Language of Many Features

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Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: Feature needs not be handled in calculus
Contra: Modified source code
Example in KeY: Very rare: treating inner classes

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- Program transformation, up-front
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Pro: Flexible, easy to implement, usable
Contra: Not expressive enough for all features
Example in KeY: Complex expression eval, method inlining, etc., etc.

Java—A Language of Many Features

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: No logic extensions required, enough to express most features
Contra: Creates difficult first-order POs, unreadable antecedents
Example in KeY: Dynamic types and branch predicates

Java—A Language of Many Features

Ways to deal with Java features

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose extensions of program logic

Pro: Arbitrarily expressive extensions possible
Contra: Increases complexity of all rules
Example in KeY: Method frames, updates

Components of the Calculus

1. Non-program rules
   - first-order rules
   - rules for data-types
   - first-order modal rules
   - induction rules
2. Rules for reducing/simplifying the program (symbolic execution)
   Replace the program by
   - case distinctions (proof branches) and
   - sequences of updates
3. Rules for handling loops
   - using loop invariants
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Loop Invariants

KIT Karlsruhe Institute of Technology

Symbolic execution of loops: unwind

$$\textsc{unwindLoop}\ \frac{\Gamma \Longrightarrow \mathcal{U}[\pi\, \mathbf{if}\,(\mathtt{b})\ \{\ \mathtt{p;}\ \mathbf{while}\,(\mathtt{b})\ \mathtt{p}\}\ \omega]\phi, \Delta}{\Gamma \Longrightarrow \mathcal{U}[\pi\, \mathbf{while}\,(\mathtt{b})\ \mathtt{p}\ \omega]\phi, \Delta}$$

How to handle a loop with...

- 0 iterations? Unwind $1\times$
- 10 iterations? Unwind $11\times$
- 10000 iterations? Unwind $10001\times$
  (and don't make any plans for the rest of the day)
- an *unknown* number of iterations?

We need an *invariant rule* (or some other form of induction)

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Loop Invariants Cont'd

Idea behind loop invariants

- A formula *Inv* that
  - holds initially and
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- *Consequence*: if *Inv* was valid at start of the loop, then it still holds after arbitrarily many loop iterations
- If the loop terminates at all, then *Inv* holds *afterwards*
- Make *Inv* strong enough to entail the desired *postcondition*

Basic Invariant Rule

$$\text{loopInvariant}\quad \frac{\begin{array}{ll} \Gamma \Longrightarrow \mathcal{U}\,\mathit{Inv}, \Delta & \text{(initially valid)} \\ \mathit{Inv}, b \doteq \mathrm{TRUE} \Longrightarrow [\mathtt{p}]\mathit{Inv} & \text{(preserved)} \\ \mathit{Inv}, b \doteq \mathrm{FALSE} \Longrightarrow [\pi\ \omega]\phi & \text{(use case)} \end{array}}{\Gamma \Longrightarrow \mathcal{U}[\pi\ \mathbf{while}\,(\mathtt{b})\ \mathtt{p}\ \omega]\phi, \Delta}$$

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Basic Invariant Rule: Problem

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Example

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int i = 0;
while(i < a.length) {
    a[i] = 1;
    i++;
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Precondition: $a \neq \textbf{null}$

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Precondition: $\mathtt{a} \neq \mathbf{null}$

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int i = 0;
while(i < a.length) {
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Keeping the Context

- Want to keep part of the context that is *unmodified* by loop
- **`assignable`** *clauses* for loops can tell what might be modified

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@ assignable i, a[*];
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Example in JML/Java – Loop.java

```
public int[] a;
/*@ public normal_behavior
  @  ensures (\forall int x; 0<=x && x<a.length; a[x]==1);
  @  diverges true;
  @*/
public void m() {
  int i = 0;
  /*@ loop_invariant
    @  (0 <= i && i <= a.length &&
    @   (\forall int x; 0<=x && x<i; a[x]==1));
    @ assignable i, a[*];
    @*/
  while(i < a.length) {
    a[i] = 1;
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  }
}
```

Example

$\forall$ `int` x;
  (n $\doteq x \wedge x >= 0 \rightarrow$
    [

```
i = 0; r = 0;
while (i<n) { i = i + 1; r = r + i;}
r=r+r-n;
```

    ]r $\doteq$?)

How can we prove that the above formula is valid (i.e., satisfied in all states)?

Solution:

```
@ loop_invariant
@     i>=0 && 2*r == i*(i + 1) && i <= n;
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```

File: `Loop2.java`

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```
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    ]$\mathtt{r} \doteq x * x$)

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Hints

Proving `assignable`

- The invariant rule on the slides *assumes* that `assignable` is correct. With `assignable \nothing;` e.g., one can prove nonsense
- The invariant rule in KeY generates *proof obligation* that ensures correctness of `assignable`

Setting in the KeY Prover when proving loops

- Loop treatment: *Invariant*
- Quantifier treatment: *No Splits with Progs*
- If program contains `*`, `/`:
  Arithmetic treatment: *DefOps*
- Is search limit high enough (time out, rule apps.)?
- When proving partial correctness, add `diverges true;`

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Total Correctness

Find a decreasing integer term v (called *variant*)

Add the following premises to the invariant rule:

- $v \geq 0$ is initially valid
- $v \geq 0$ is preserved by the loop body
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Proving termination in JML/Java

- Remove directive `diverges true;`
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Files:

- `LoopT.java`
- `Loop2T.java`

Side effects in loop guards

Find a postcondition:

```
int x, y;
// ...
while( x-- != ++y );
```

Note: Loop guards may have side effects.
Hence: Evaluate them in a modality.

Invariant rule with side effects

$$\text{sideEffectLI}\ \frac{\begin{array}{ll}\Gamma \Longrightarrow \mathcal{U}\, Inv, \Delta & \text{(initia} \\ Inv, b \doteq \mathrm{TRUE} < 4 > [\mathtt{x=b;}] x \doteq \mathrm{TRUE} \Longrightarrow [p] Inv & \text{(pres} \\ Inv, b \doteq \mathrm{FALSE} \Longrightarrow [\pi\ \omega]\phi & \text{(use}\end{array}}{\Gamma \Longrightarrow \mathcal{U}[\pi\ \mathtt{while}(\mathtt{b})\ \mathtt{p}\ \omega]\phi, \Delta}$$

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Loops and Abrupt Completion

Rule loopInvariant requires normal, structural control flow
(loop body always fully executed; run continues after loop)

Non-structural control flow in Java

`return` `break` `continue` `throw`

make loop body terminate abruptly.

Solution

Transform non-standard control flow into standard control-flow and with marker variables.

Loops and Abrupt Completion

Original loop body p

```
if(x == 0) break;

if(x == 1) return 42;


if(x == 2) continue;
if(x == 3) throw e;
if(x == 4) x = -1;
```

Encoded loop body $\widehat{p}$

```
loopBody: { try {
  BREAK = RETURN = false;
  EXCEPTION = null;
  if(x == 0) { BREAK=true;
    break loopBody; }
  if(x == 1) { res=42;
    RETURN=true;
    break loopBody; }
  if(x == 2) break loopBody;
  if(x == 3) throw e;
  if(x == 4) x = -1;
} catch(Throwable e) { EXC = e; }}
```

Loops and Abrupt Completion

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Invariant rule with abrupt termination (using translation $\widehat{\cdot}$)

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where ψ is the formula

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Loop Invariant – Conclusion

Is a difficult subject.
shows that real prog language is a challenge
Many technical non-trivial tricks.
A rule that puts together

1. considering `assignable` clauses
2. side effects in loop guards
3. abrupt termination

is in chapter 3.
Further reading: KeY book Ch. 15 ??
New developments: Loop scope rule, Loop contracts