PdF project:
Pattern Dominating Set in Sparse Graphs

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Background

Many graph problems can be solved faster when the given graph is *sparse*, i.e., contains few edges. A recent work [1] conditionally settles the time complexity of the famous k-dominating set problem in sparse graphs. In particular (for sufficiently large k), the time complexity reduces from $n^{k\pm o(1)}$ in dense graphs to $n^{k-1\pm o(1)}$ in graphs with only $m = O(n)$ edges – this is tight assuming the Strong Exponential Time Hypothesis (SETH).

Subsequent work [2] studies the fine-grained complexity of variants of k-dominating set in which the dominating set S must adhere to a given k-node pattern H, i.e., the subgraph induced by S is equal to H. If H is the perfect matching on k nodes, this is closely related to the *paired dominating set* problem and [2] establishes a time complexity of $m^{k/2\pm o(1)}$ under SETH - i.e., when $m = O(n)$, the exponent is halved compared to dense graphs! For H being a clique or an independent set, the time complexity reduces even more, albeit slightly.

Objective

While [2] shows that the time complexity of H-pattern k-dominating set depends on the pattern H, it only studies these three patterns: perfect matching, clique, and independent set. The task is to extend the techniques of [1, 2] to further patterns H, ideally determining the tight time complexity for all H under SETH.

A possible extension is to experimentally evaluate the resulting algorithms for H-pattern detection algorithms (including those given in [1, 2]).

References

[1] N. Fischer, M. Künnemann, and M. Redzic. The effect of sparsity on k-dominating set and related first-order graph properties. In *Proc. 2024 ACM-SIAM Symposium on Discrete Algorithms (SODA 2024)*, pages 4704–4727. SIAM, 2024.

[2] M. Künnemann and M. Redzic. Fine-grained complexity of multiple domination and pattern domination in sparse graphs. Unpublished manuscript, 2024.