Institute of Information Security and Dependability (KASTEL)
Application-oriented Formal Verification
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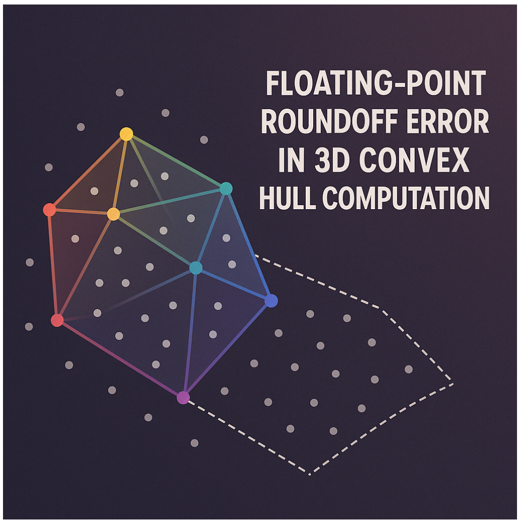
Praxis der Forschung: Computing Reliable 3-D Convex Hulls with Finite Precision



Context Convex hulls are fundamental structures in computational geometry, widely used in computer graphics, collision detection, and scientific computing. Although there exist well-established algorithms for constructing convex hulls, they are typically described under the assumption of exact real arithmetic. In practice, however, these algorithms are implemented using floating-point arithmetic, due to the finite precision and memory limitations of digital computers. Core operations such as dot products and orientation tests in floating-point computations are inherently imprecise and susceptible to roundoff errors. These numerical inaccuracies can accumulate and lead to incorrect decisions about whether points lie inside or outside the hull. As a result, the computed structure may not be truly convex, severely compromising its geometric integrity and introducing fatal inconsistencies in downstream applications [1].

State-of-the-Art: While the correctness of convex hull algorithms has been studied under the assumption of real arithmetic [2], research has shown that finite-precision implementations can introduce roundoff errors that compromise the correctness of the resulting hulls [1]. Despite these findings, there has been limited work on providing formal soundness guarantees in the presence of floating-point errors during convex hull computation. Meanwhile, tools, such as Daisy [3], have been developed to compute sound roundoff error bounds for straight-line floating-point programs. These tools have not yet been applied in the context of computational geometry, where their use could provide valuable guarantees about numerical robustness.

Goal of the Project This project explores the impact of floating-point roundoff errors on the geometric correctness of 3D convex hull computations and to develop an approximate version of the convex hull algorithm that minimizes deviation from the ideal, real-valued result. The overarching goal is to strike a practical balance between computational efficiency and numerical robustness, ensuring that the computed hull remains both accurate and structurally sound despite inherent limitations of finite precision.

Your Profile There are no strict prerequisites for the project. However, prior experience with computational geometry, program analysis, verification, and familiarity with Scala or Java would be helpful.

References

[1] Bernhard Beckert, Britta Nestler, Moritz Kiefer, Michael Selzer, and Mattias Ulbrich. Experience report: Formal methods in material science. *arXiv preprint arXiv:1802.02374*, 2018.

[2] Christophe Brun, Jean-François Dufourd, and Nicolas Magaud. Designing and proving correct a convex hull algorithm with hypermaps in coq. *Computational Geometry*, 45(8):436–457, 2012.

[3] Eva Darulova, Anastasiia Izycheva, Fariha Nasir, Fabian Ritter, Heiko Becker, and Robert Bastian. Daisy - framework for analysis and optimization of numerical programs (tool paper). In *Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*, 2018.

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