Reconfiguration of Graphs

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Given a set of combinatorial objects $\mathcal{U}$ and a set of operations $\mathcal{F}$ on these objects, the *reconfiguration graph* $\mathcal{R}_{\mathcal{F}}(\mathcal{U})$ is defined as the graph with vertex-set $\mathcal{U}$, i. e., the combinatorial objects in $\mathcal{U}$ are vertices of $\mathcal{R}_{\mathcal{F}}(\mathcal{U})$, and an edge between two vertices $U_1, U_2 \in \mathcal{U}$ if and only if there is an operation $f \in \mathcal{F}$ such that $f(U_1) = U_2$. The properties of reconfiguration graphs have been extensively studied for various combinatorial objects, such as triangulations in the plane, spanning trees of fixed point sets in the plane, as well as drawings, independent sets, or colorings of a fixed graph, just to name a few.

However, little is known for the case that $\mathcal{U}$ is a class of graphs. We want to study the operations $\mathcal{F}$ of adding or removing a single edge from a graph in $\mathcal{U}$, and investigate properties of the resulting reconfiguration graph. That is, for different graph classes $\mathcal{G}$, we want to examine the graph $\mathcal{R}_{\mathcal{F}}(\mathcal{G})$ with vertex-set $\mathcal{G}$, and an edge between two vertices $G_1, G_2 \in \mathcal{G}$ if and only if G_2 can be obtained by adding an edge to G_1 or deleting an edge from G_1. Possible candidates for the graph class $\mathcal{G}$ include chordal graphs, perfect graphs, interval graphs, comparability graphs, as well as graphs of treewidth k or treedepth k for some fixed $k \in \mathbb{N}$.

We are in particular interested in properties of the resulting reconfiguration graphs that are frequently considered in this field. This includes the connectivity, i. e., whether the reconfiguration graph is connected and if not how many components it has, the diameter and radius, as well as the computational complexity to compute a shortest path between two given vertices.