
KI-Programmierung

Informed Search

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Outline

- **Best-first search**

- **A* search**

- **Heuristics**

Review: Tree search

function TREE-SEARCH(*problem*, *fringe*) **returns** a solution or failure

fringe $\leftarrow$ INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)

loop do

if *fringe* is empty **then return** failure

node $\leftarrow$ REMOVE-FIRST(*fringe*)

if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **then**

return *node*

else

fringe $\leftarrow$ INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

end

Strategy

Defines the order of node expansion

Best-first search

Idea

Use an **evaluation function** for each node (estimate of “desirability”)

Expand most desirable unexpanded node

Implementation

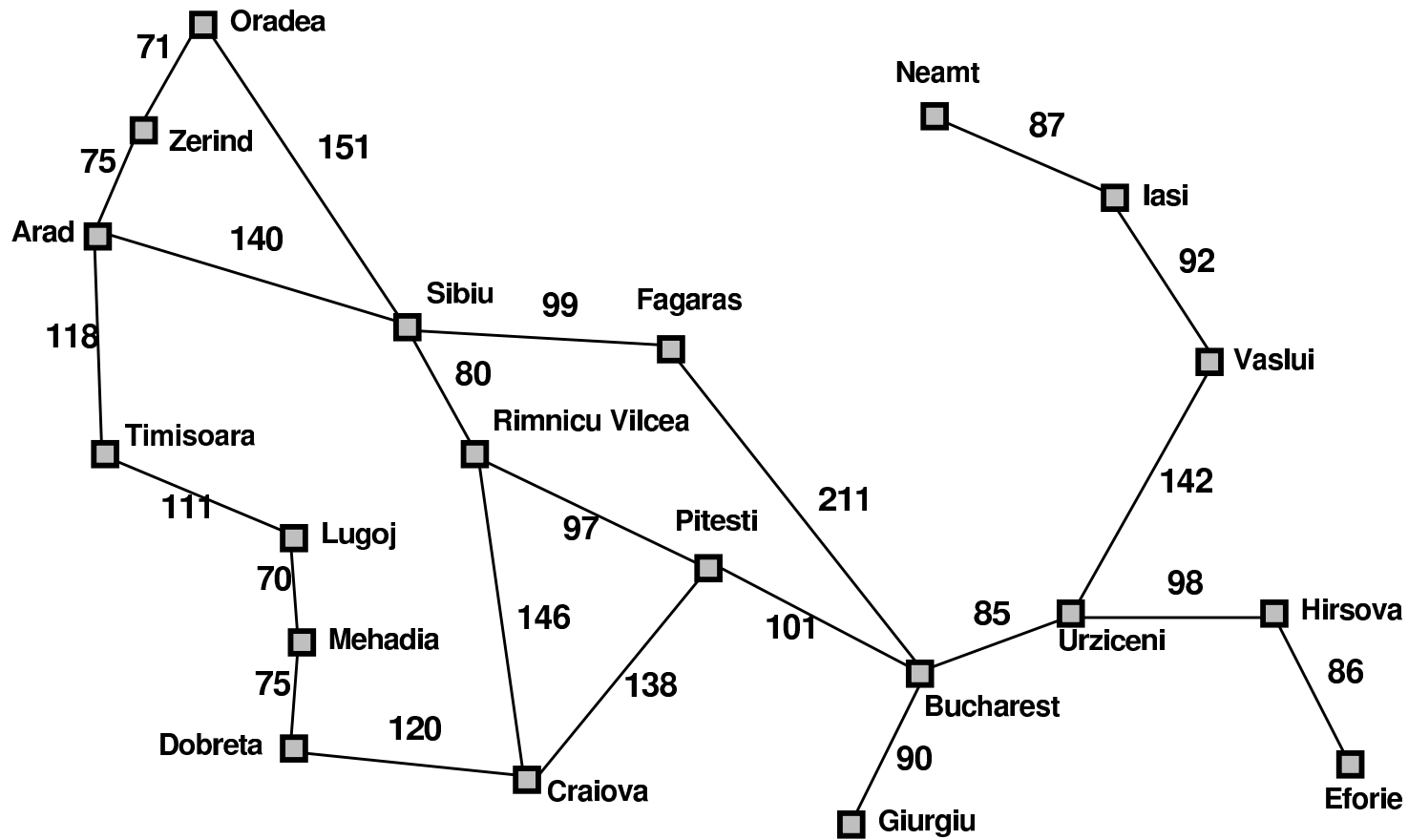
fringe is a queue sorted in decreasing order of desirability

Special cases

- 🟡 Greedy search

- 🟡 A* search

Romania with step costs in km



Straight-line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

Heuristic

Evaluation function

$$h(n) = \text{estimate of cost from } n \text{ to } goal$$

Greedy search expands the node that **appears** to be closest to goal

Example

$$h_{\text{SLD}}(n) = \text{straight-line distance from } n \text{ to Bucharest}$$

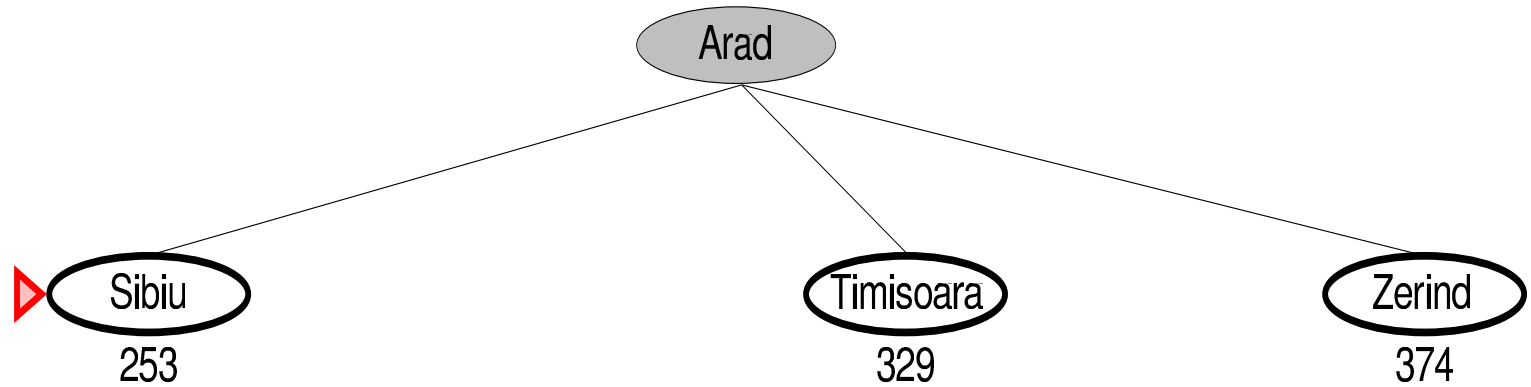
Note

Unlike uniform-cost search the node evaluation function has nothing to do with the nodes explored so far

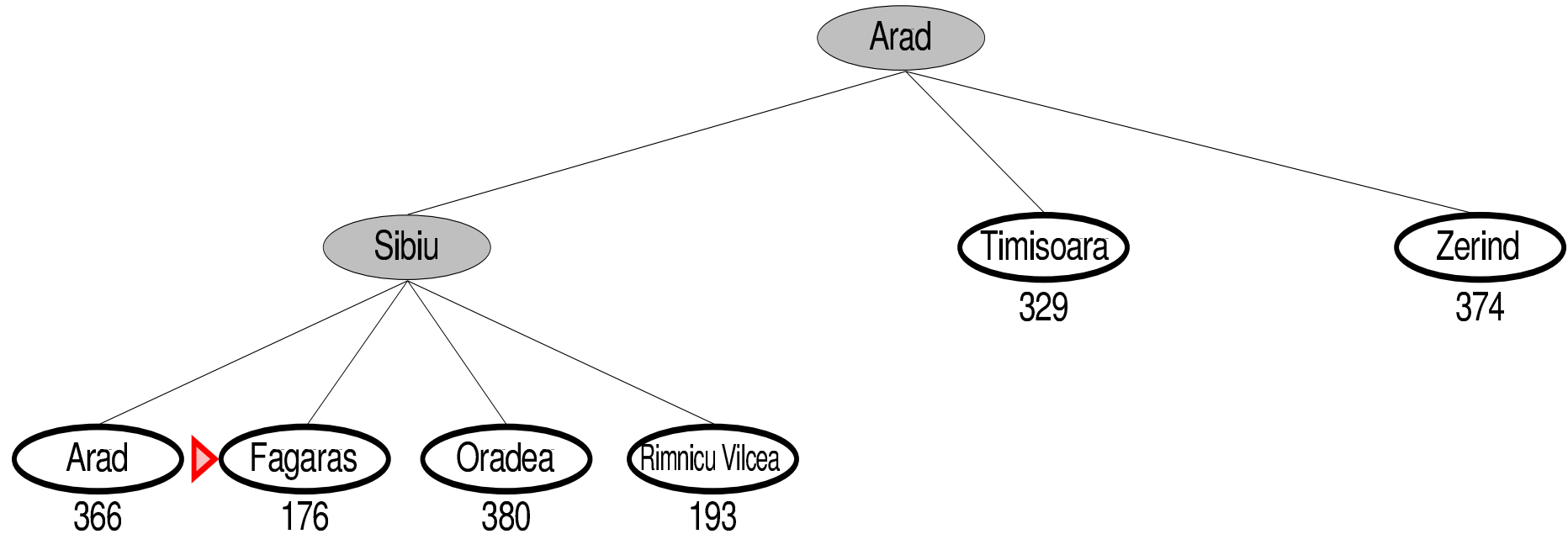
Greedy search: Example Romania

▶ Arad
366

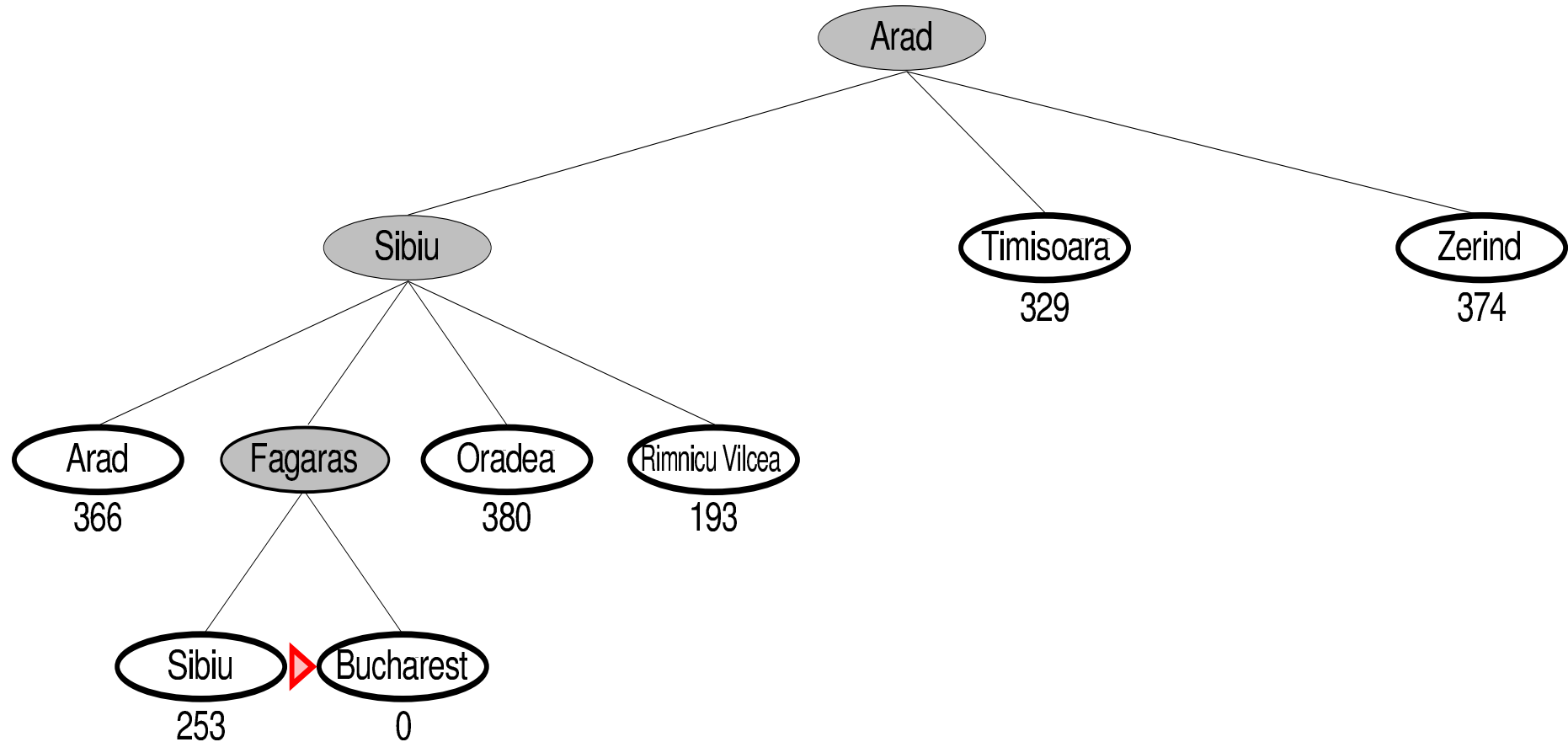
Greedy search: Example Romania



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Greedy search: Example Romania



Greedy search: Properties

Complete

Time

Space

Optimal

Greedy search: Properties

Complete **No**

Can get stuck in loops

Example: Iasi to Oradea

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$...

Complete in finite space with repeated-state checking

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Space

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$O(b^m)$

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Complete in finite space with repeated-state checking

Time $O(b^m)$

Space $O(b^m)$

Optimal **No**

Note

Worst-case time same as depth-first search,

Worst-case space same as breadth-first

But a good heuristic can give dramatic improvement

A* search

Idea

Avoid expanding paths that are already expensive

Evaluation function

$$f(n) = g(n) + h(n)$$

where

$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost to goal from n

$f(n)$ = estimated total cost of path through n to goal

A* search: Admissibility

Admissibility of heuristic

$h(n)$ is admissible if

$$h(n) \leq h^*(n) \quad \text{for all } n$$

where $h^*(n)$ is the **true** cost from n to goal

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Also required

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In particular: $h(G) = 0$ for goal G

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Example

Straight-line distance never overestimates the actual road distance

A^* search: Admissibility

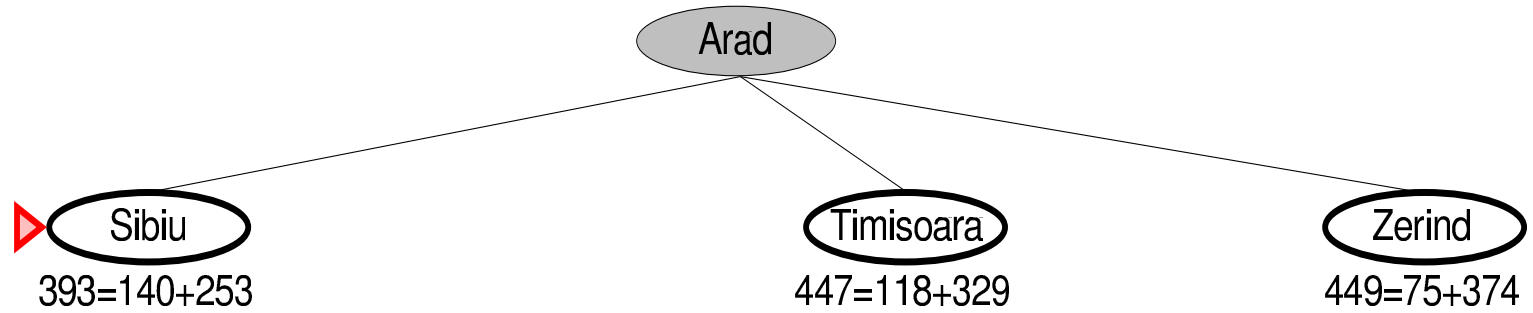
Theorem

A^* search with admissible heuristic is optimal

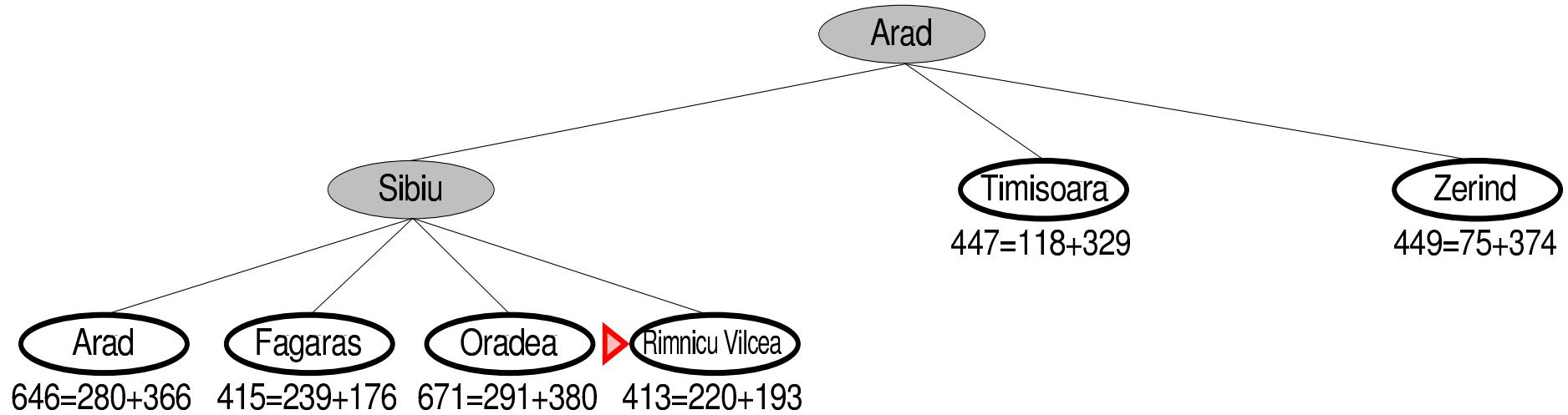
A* search example

▶ Arad
366=0+366

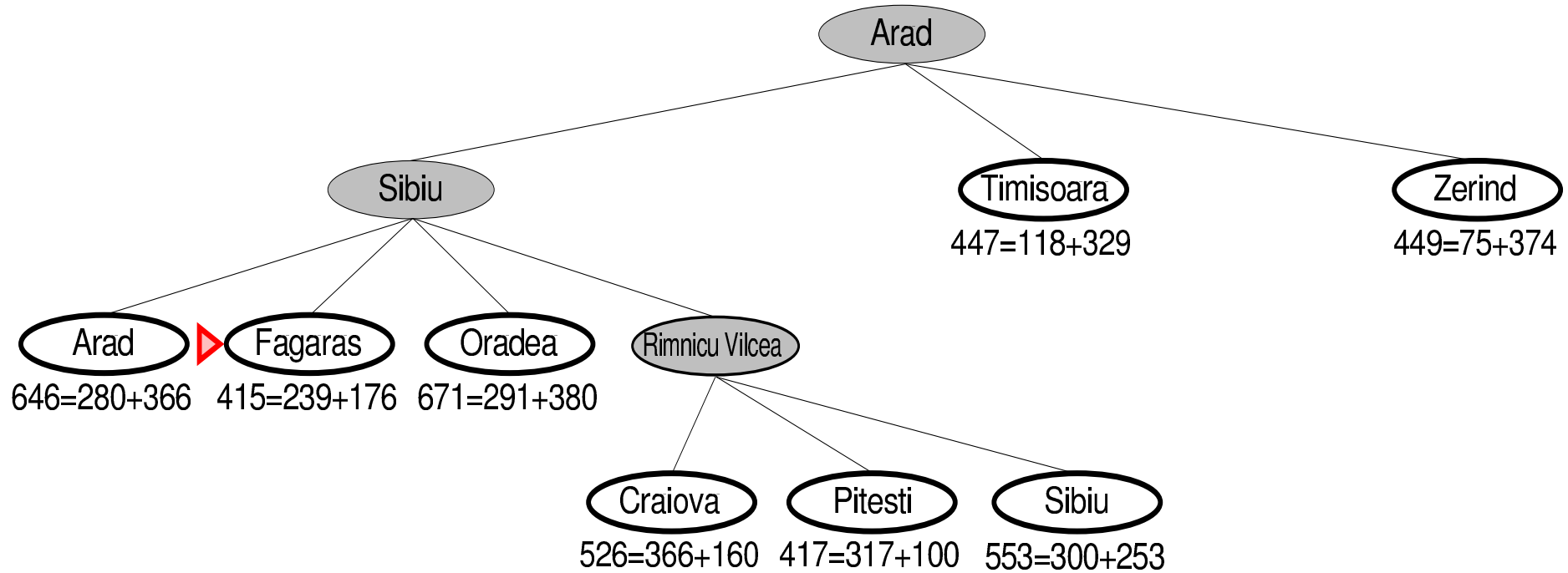
A* search example



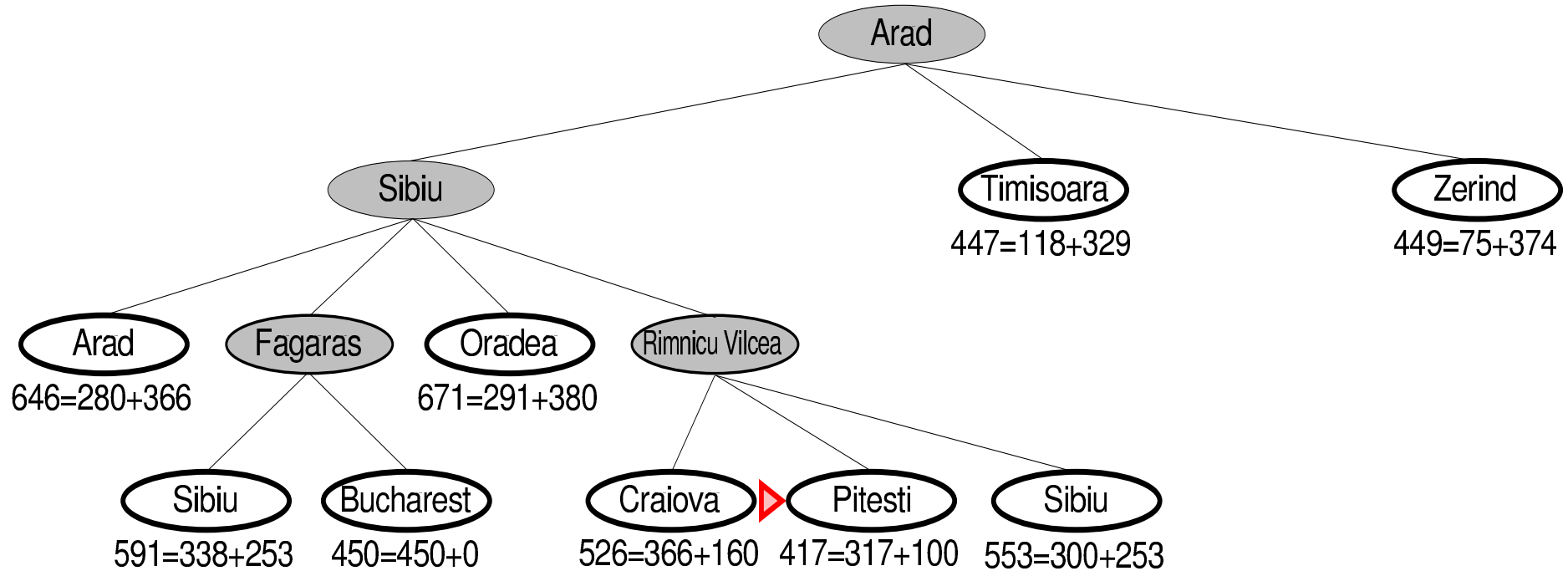
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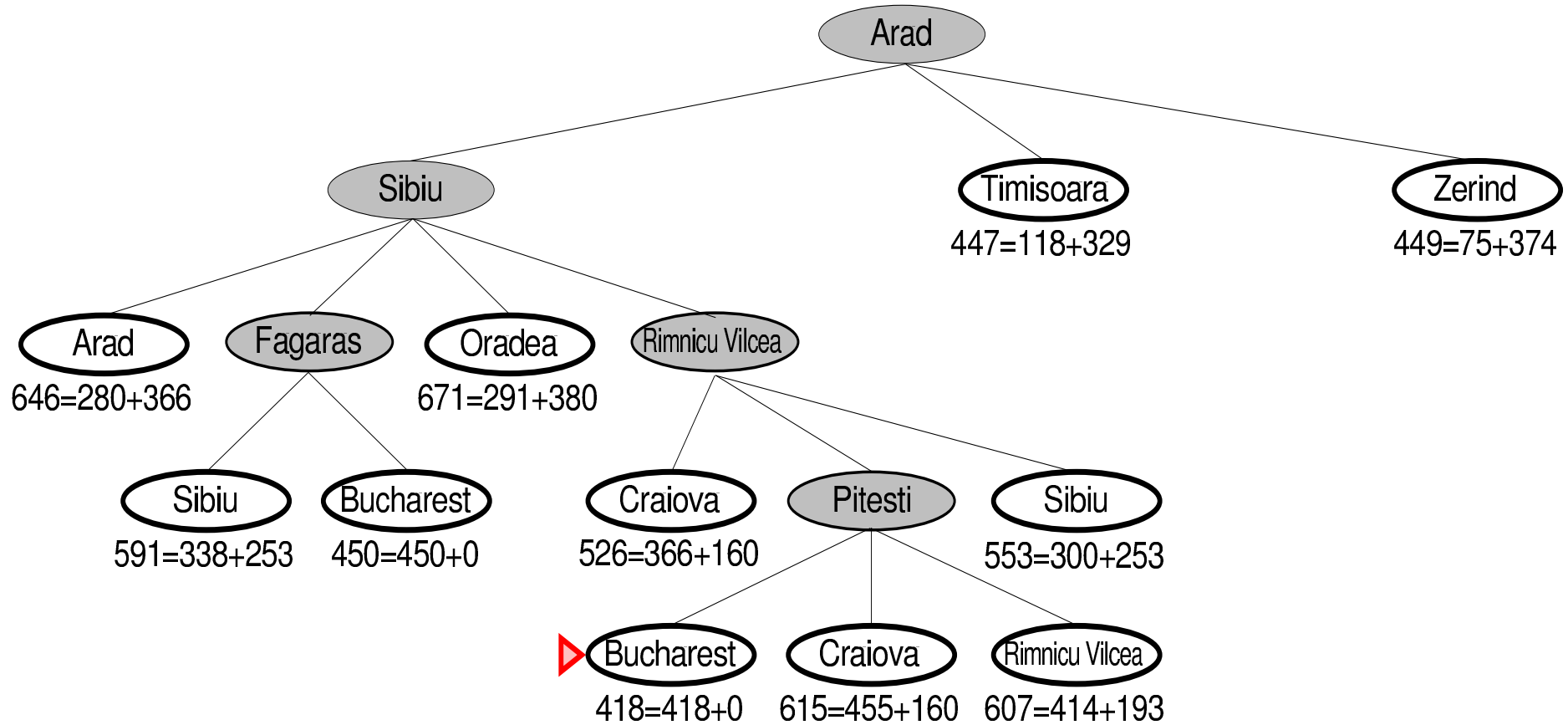
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A* search example

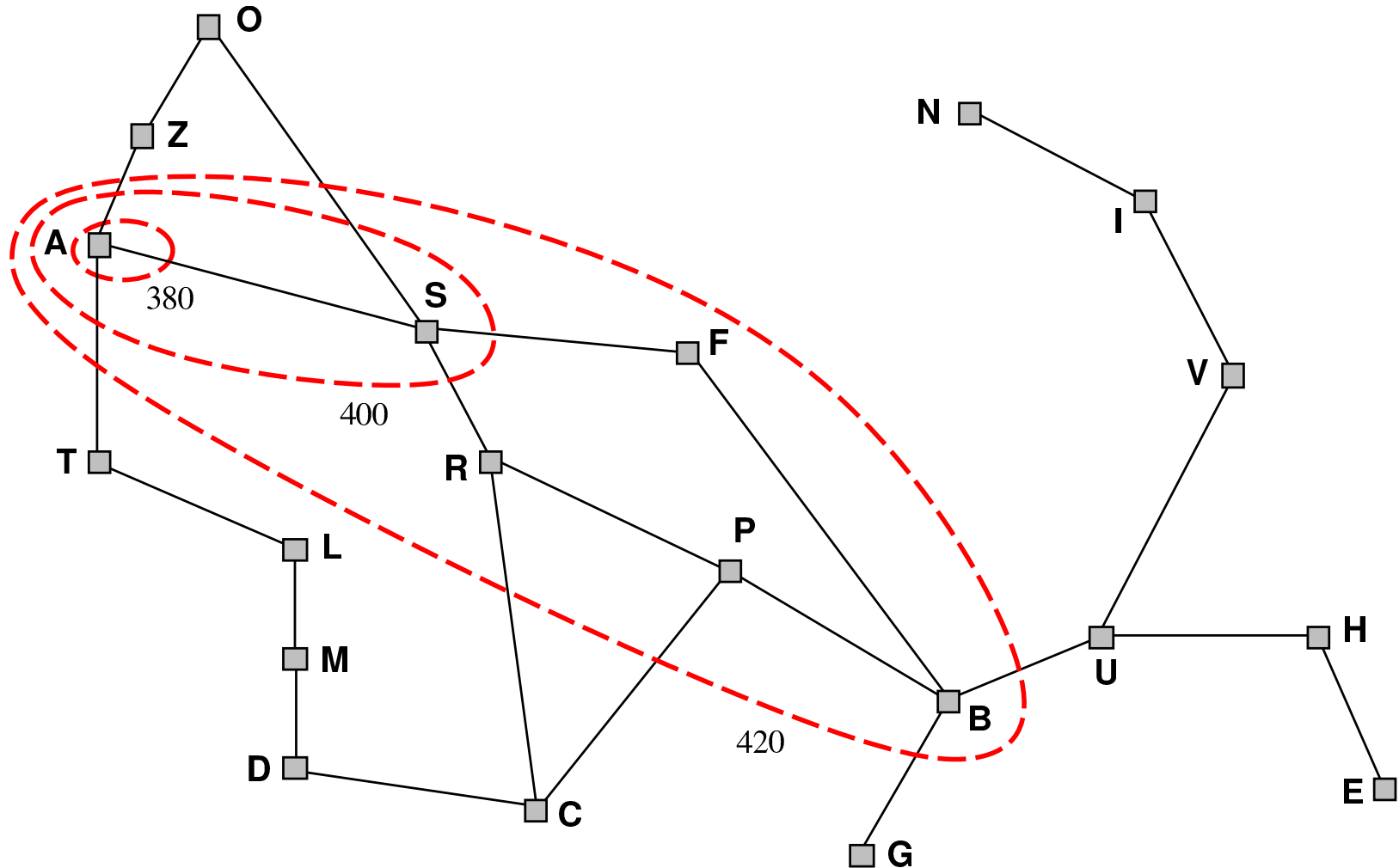


A* search example

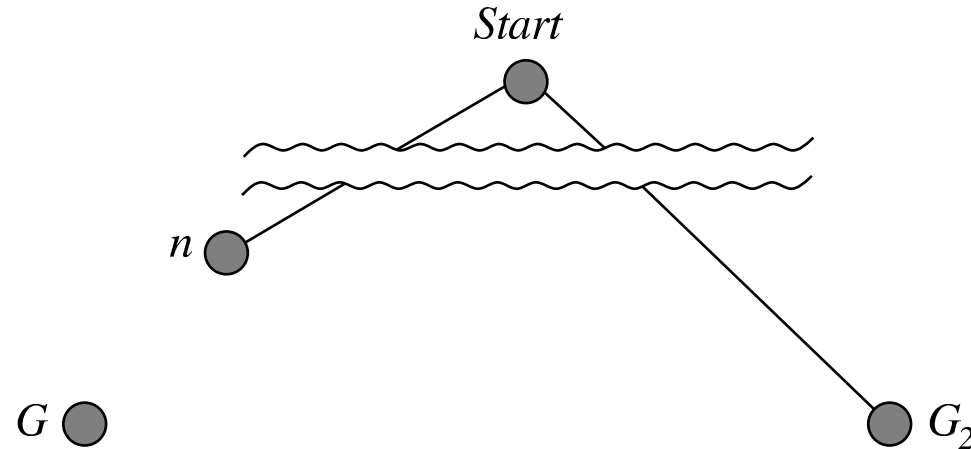


A^* search: f -contours

A^* gradually adds “ f -contours” of nodes



Optimality of A* search: Proof



Suppose a suboptimal goal G_2 has been generated

Let n be an unexpanded node on a shortest path to an optimal goal G

$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G) && \text{since } G_2 \text{ suboptimal} \\ &= g(n) + h^*(n) \\ &\geq g(n) + h(n) && \text{since } h \text{ is admissible} \\ &= f(n) \end{aligned}$$

Thus, A* never selects G_2 for expansion

A* search: Properties

Complete

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Optimal

A* search: Properties

Complete **Yes**

(unless there are infinitely many nodes n with $f(n) \leq f(G)$)

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[relative error in $h \times$ length of solution]

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[relative error in $h \times$ length of solution]

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Note

A* expands all nodes with $f(n) < C^*$

A* expands some nodes with $f(n) = C^*$

A* expands no nodes with $f(n) > C^*$

Admissible heuristics: Example 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

Admissible heuristics

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

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In the example

$$h_1(S) = 6$$

$$h_2(S) = 2 + 0 + 3 + 1 + 0 + 1 + 3 + 4 = 14$$

Dominance

Definition

h_1, h_2 two admissible heuristics

h_2 **dominates** h_1 if

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Theorem

If h_2 dominates h_1 , then h_2 is better for search than h_1 .

Dominance: Example 8-puzzle

Typical search costs

$d = 14$ **IDS** **3,473,941 nodes**

$A^*(h_1)$ **539 nodes**

$A^*(h_2)$ **113 nodes**

$d = 24$ **IDS** **too many nodes**

$A^*(h_1)$ **39,135 nodes**

$A^*(h_2)$ **1,641 nodes**

d : **depth of first solution**

IDS: **iterative deepening search**

Relaxed problems

Finding good admissible heuristics is an art!

Deriving admissible heuristics

Admissible heuristics can be derived from the exact solution cost of a *relaxed* version of the problem

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If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then we get heuristic h_1

If the rules are relaxed so that a tile can move to *any adjacent square*, then we get heuristic h_2

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Key point

The optimal solution cost of a relaxed problem is not greater than the optimal solution cost of the real problem