Modale Ableitbarkeit

Ein modallogisches System besteht aus Axiomen und Schlußregeln

Ein modallogische System $\mathbf{S}$ induziert eine Ableitbarkeitsrelation $\vdash_{\mathbf{S}}$.

$$\Sigma \vdash_{\mathbf{S}} A$$

gilt für eine Formelmenge $\Sigma \subseteq Fml_{ALmod}$ und eine Formel $A \in Fml_{ALmod}$, wenn es einen Beweis für A gibt, der nur die Formeln aus Σ und die Axiome und Schlußregeln von $\mathbf{S}$ benutzt.

Das modallogische System $\mathbf{K}$

Axiome alle AL-Tautologien

$$(A \to B) \to (\Box(A) \to \Box B) \qquad \text{(K)}$$

Regeln

$$\frac{A\,,\,A \to B}{B} \qquad \text{(MP)}$$

$$\frac{A}{\Box A} \qquad \text{(G)}$$

Einige modallogische Systeme

T	**K**	+	$\Box(A) \rightarrow A$
D	**K**	+	$\Box(A) \rightarrow \Diamond A$
B	**T**	+	$\neg A \rightarrow \Box\neg\Box A$
S4	**T**	+	$\Box(A) \rightarrow \Box\Box A$
S5	**T**	+	$\neg\Box(A) \rightarrow \Box\neg\Box A$
S4.2	**S4**	+	$\Diamond\Box(A) \rightarrow \Box\Diamond A$
S4.3	**S4**	+	$\Box(\Box(A \rightarrow B)) \vee \Box(\Box(B \rightarrow A))$
C	**K**	+	$\dfrac{A \rightarrow B}{\Box(A \rightarrow B)}$ statt (G).

Kripke Frames and Kripke Structures

Definition

A Kripke frame

$$\mathcal{F} = (S, R)$$

consists of

- a non-empty set S (of worlds / states)
- an *accessibility relation* $R \subseteq S \times S$

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Definition

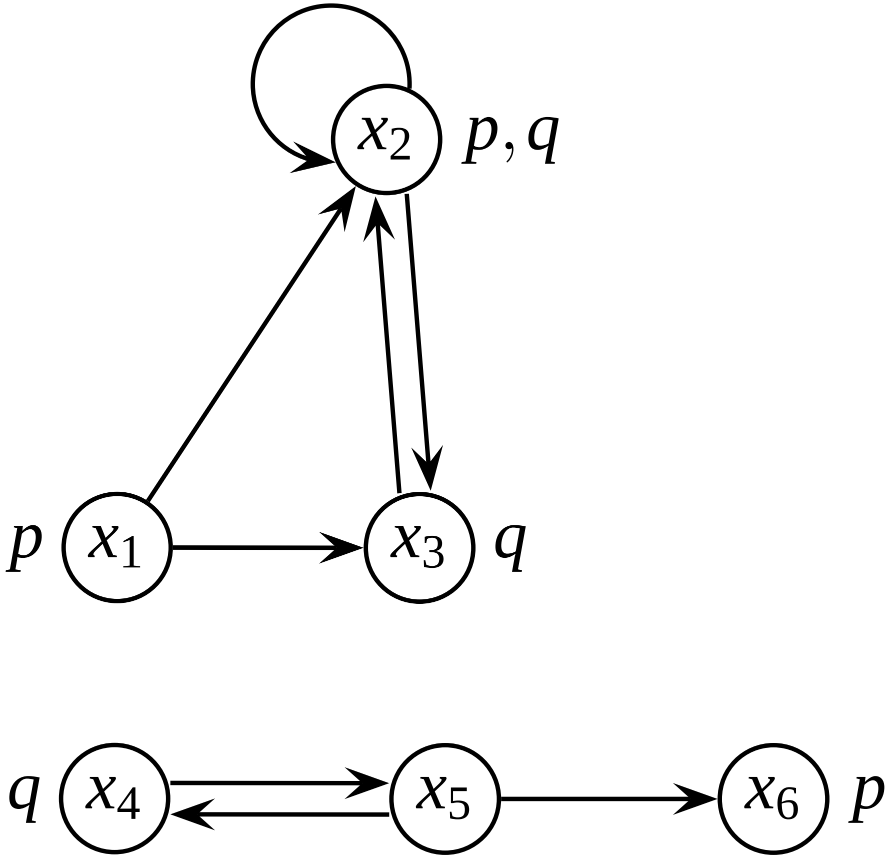
A Kripke structure

$$\mathcal{K} = (S, R, I)$$

consists of

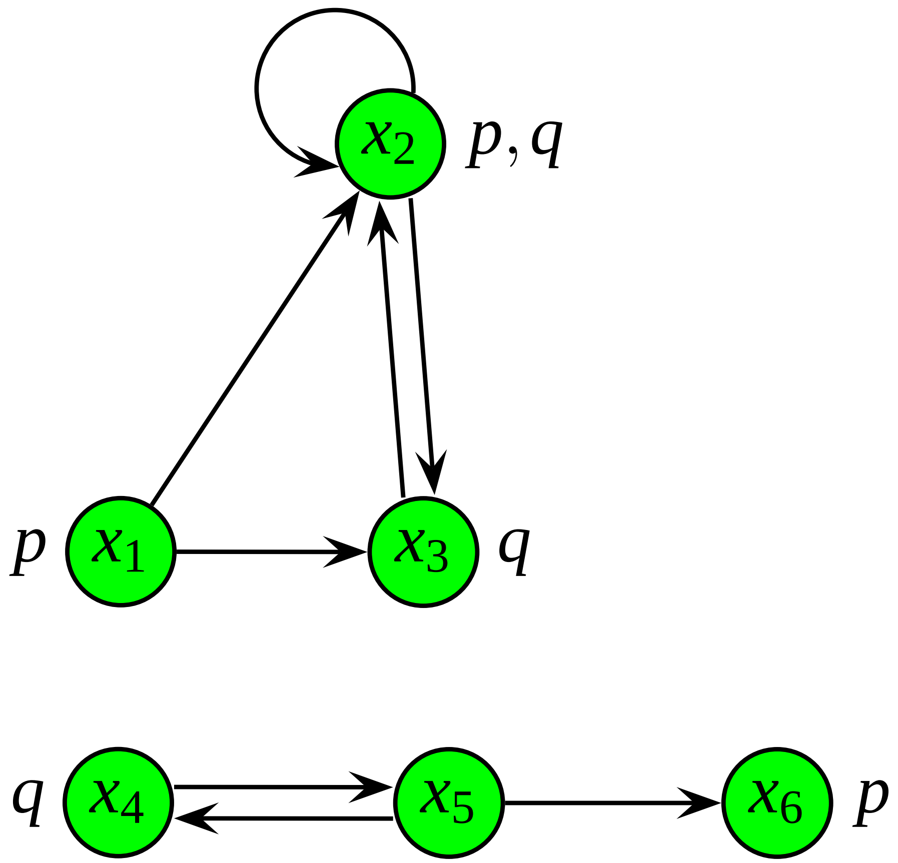
- a Kripke frame $\mathcal{F} = (S, R)$
- an *interpretation* $I : ALVar \times S \to \{1, 0\}$

Kripke Structures: Example



Kripke Structures: Example

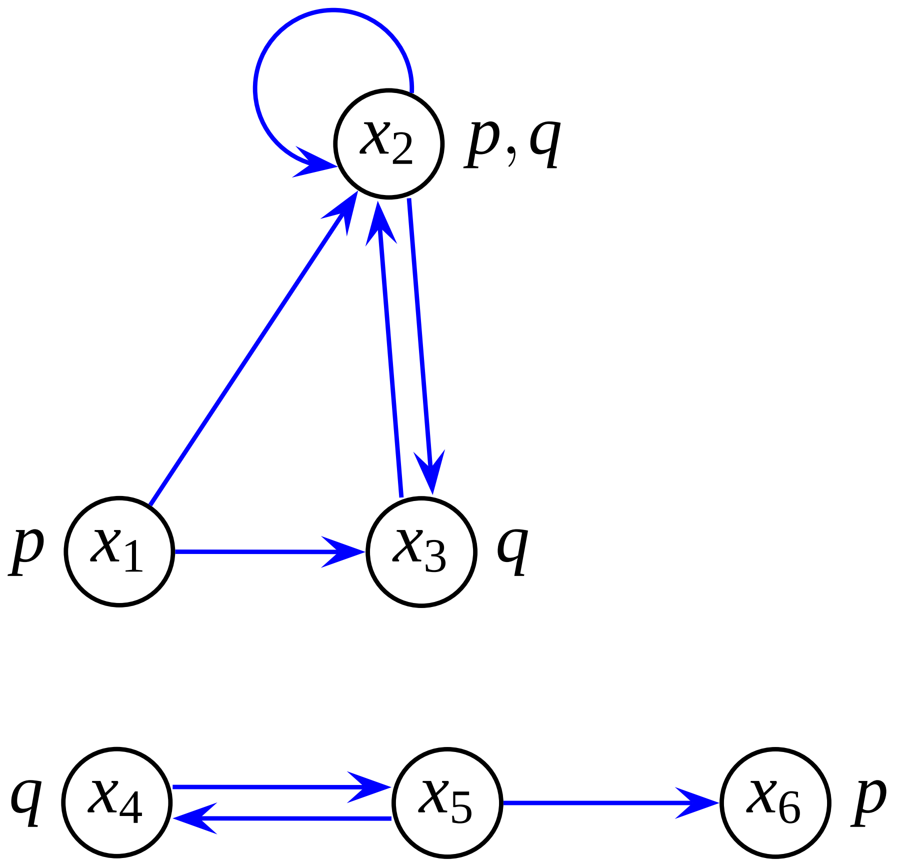
set of states



Kripke Structures: Example

accessibility relation

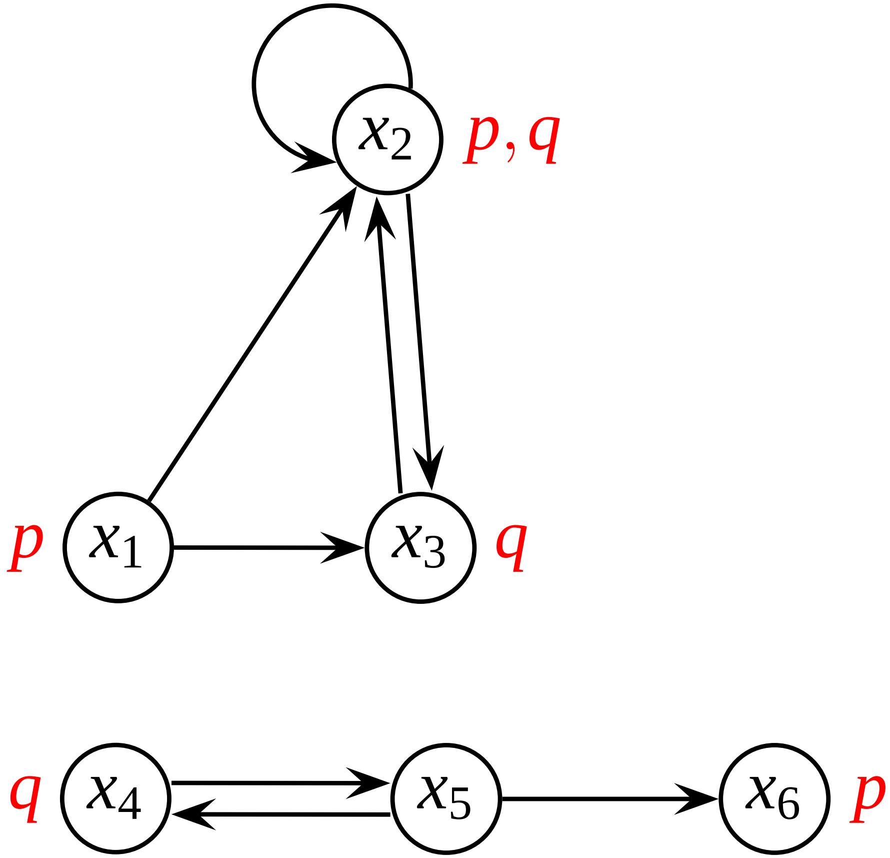
set of states



Kripke Structures: Example

accessibility relation

set of states



Interpretation I

Modal Logic: Semantics

Given: Kripke structure $\mathcal{K} = (S, R, I)$

Valuation

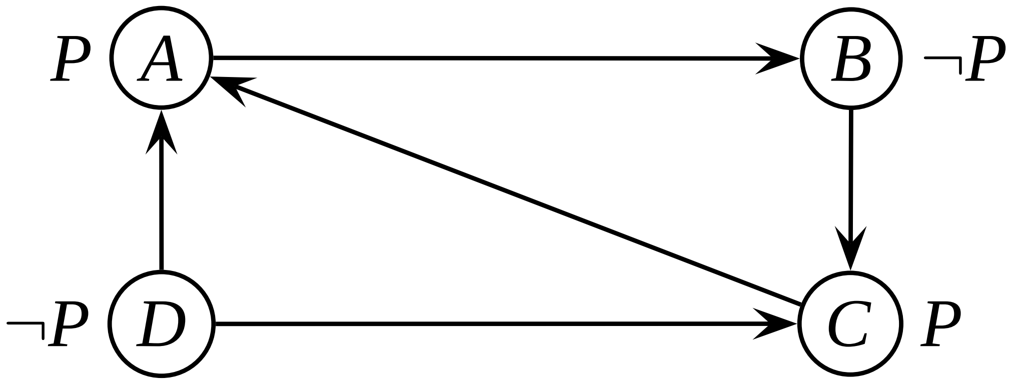
$val_{\mathcal{K}}(p)(s) \; = \; I(p)(s) \qquad$ for $p \in ALVar$

$val_{\mathcal{K}}$ defined for propositional operators in the same way as in classical logic

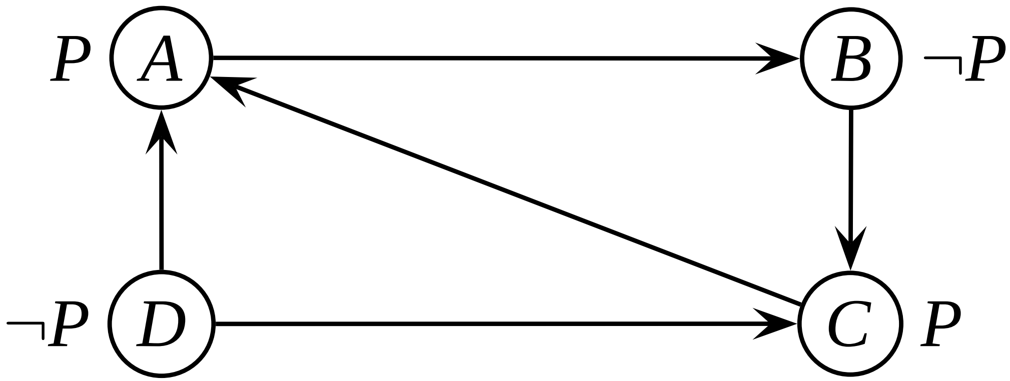
$$val_{\mathcal{K}}(\Box A)(s) \; = \; \begin{cases} 1 & \text{if } val_{\mathcal{K}}(A)(s') = 1 \text{ for} \\ & \text{all } s' \in S \text{ with } sRs' \\ 0 & \text{otherwise} \end{cases}$$

$$val_{\mathcal{K}}(\Diamond A)(s) \; = \; \begin{cases} 1 & \text{if } val_{\mathcal{K}}(A)(s') = 1 \text{ for} \\ & \text{at least one } s' \in S \text{ with } sRs' \\ 0 & \text{otherwise} \end{cases}$$

Modal Logic: Example for Evaluation



Modal Logic: Example for Evaluation



$(\mathcal{K}, A) \models P$	$(\mathcal{K}, B) \models \neg P$	$(\mathcal{K}, C) \models P$	$(\mathcal{K}, D) \models \neg P$
$(\mathcal{K}, A) \models \Box \neg P$	$(\mathcal{K}, B) \models \Box P$	$(\mathcal{K}, C) \models \Box P$	$(\mathcal{K}, D) \models \Box P$
$(\mathcal{K}, A) \models \Box \Box P$	$(\mathcal{K}, B) \models \Box \Box P$	$(\mathcal{K}, C) \models \Box \Box \neg P$	—

Saul Aaron Kripke

Born 1940 in Omaha (US)	
First publication:	*A Completeness Theorem in Modal Logic* The Journal of Symbolic Logic, 1959
Studied at:	Harvard, Princeton, Oxford and Rockefeller University
Positions:	Harvard, Rockefeller, Columbia, Cornell, Berkeley, UCLA, Oxford
since 1977	Professor at Princeton University
since 1998	Emeritus at Princeton University