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**Formal Specification and Verification of Software**

# **UML Class Diagrams by Example**

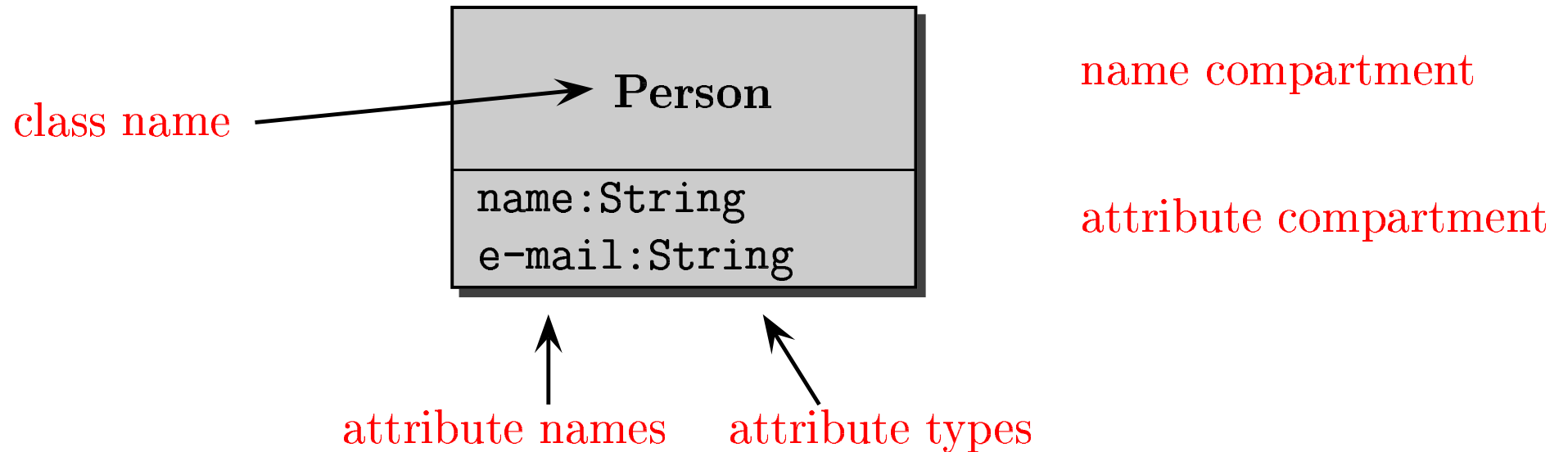
**Bernhard Beckert**



**UNIVERSITÄT KOBLENZ-LANDAU**

# Classes and Attributes

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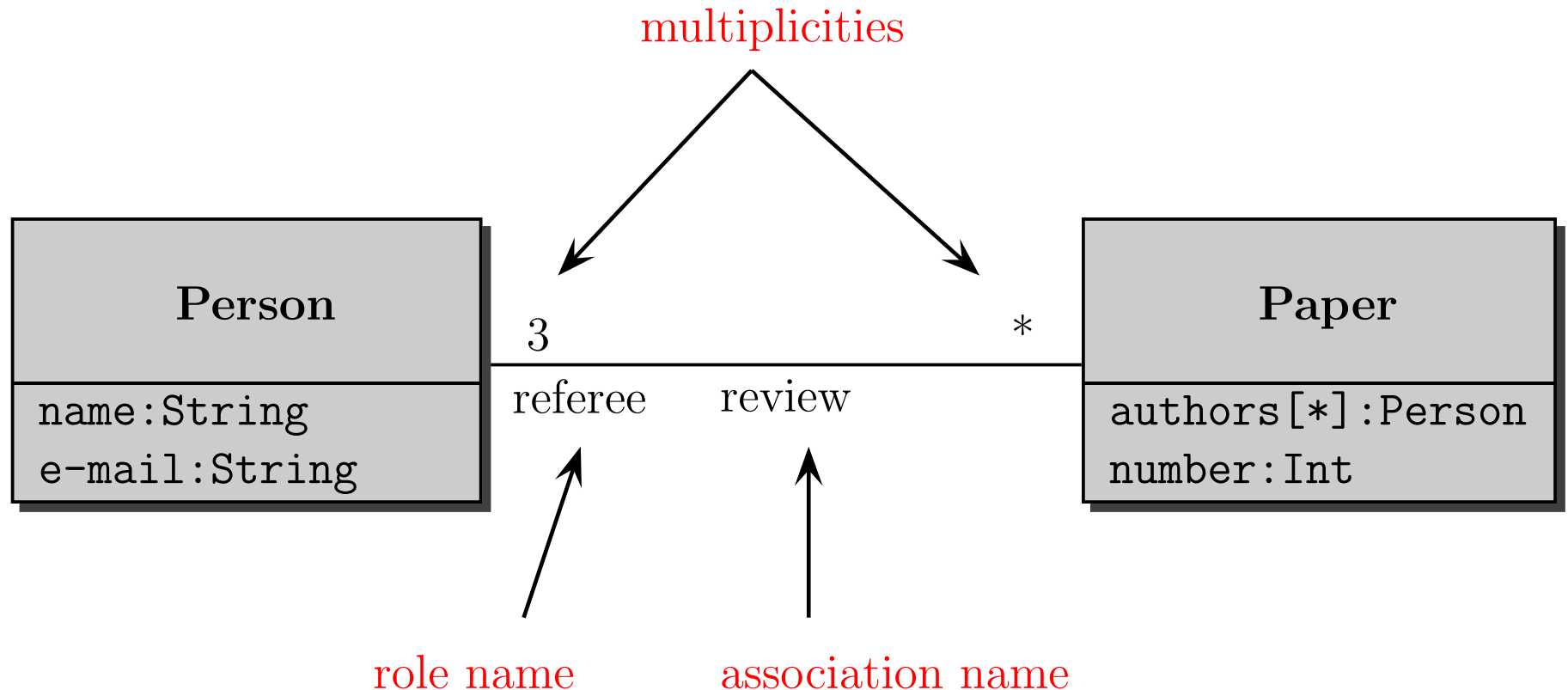


## Semantics

$I(\text{Person})$  is a (possibly empty) set

$I(\text{name})$  is a (partial) function from  $I(\text{Person})$  to  $I(\text{String})$

# Associations



## Semantics

$I(\text{review})$  is a relation between  $I(\text{Person})$  and  $I(\text{Paper})$

**Multiplicity 3** requires that, for all  $pap \in I(\text{Paper})$ ,  
 $\text{card}(\{\text{pers} \in I(\text{Person}) \mid \text{review}(\text{pers}, \text{pap})\}) \in I(3) = \{3\}$

# Multiplicities

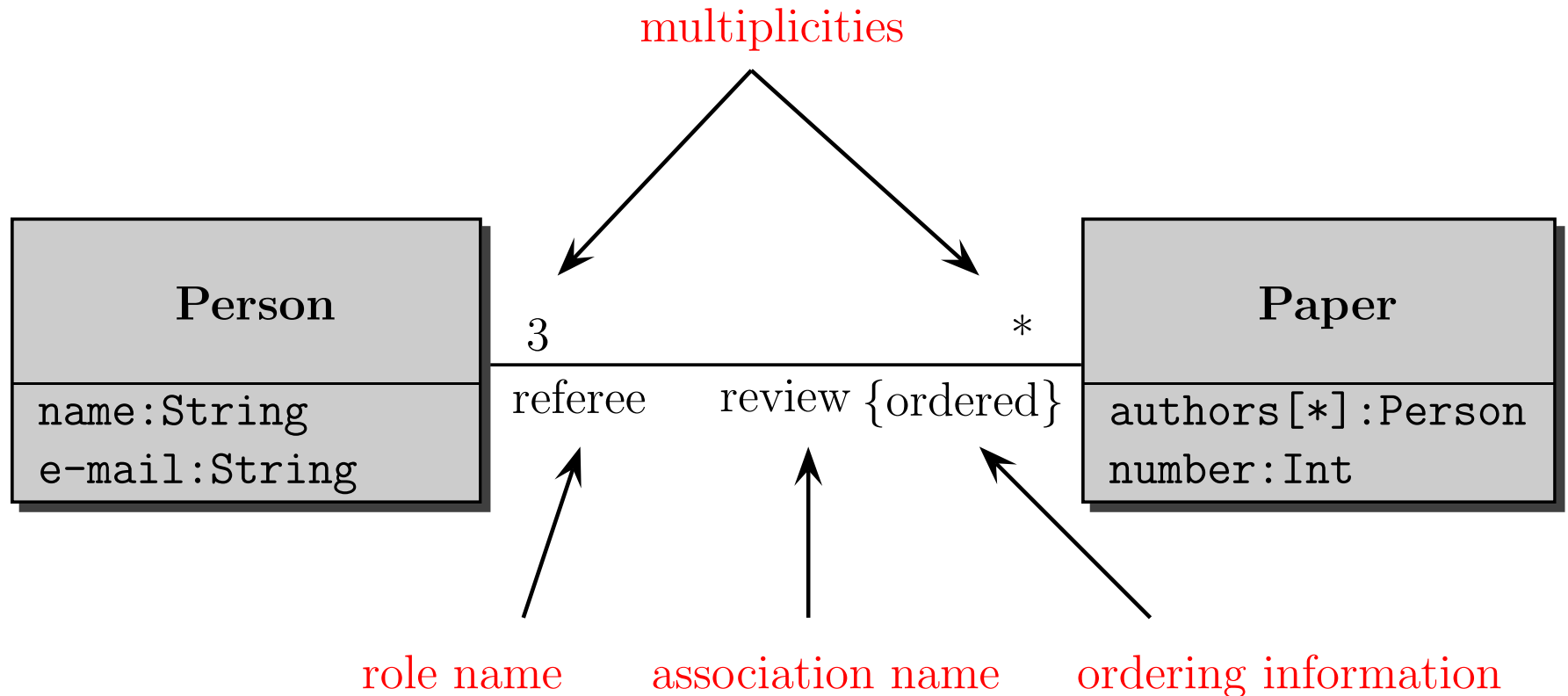
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## Semantics

$M$	$I(M)$
0..1	$\{0, 1\}$
0..*	$\mathbb{N}$
*	$\mathbb{N}$
1..3	$\{1, 2, 3\}$
7	$\{7\}$
15..19	$\{15, 16, 17, 18, 19\}$
1..3, 7, 15..19	$\{1, 2, 3, 7, 15, 16, 17, 18, 19\}$

**(i.e., the separator "," acts as set theoretic union)**

# Role Names



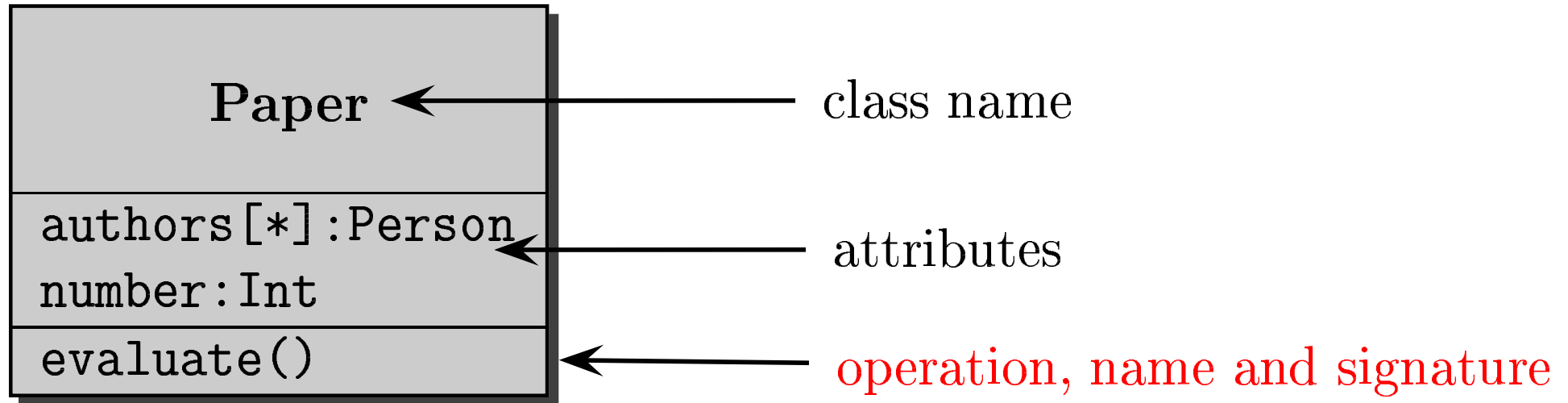
## Semantics

$I(\text{referee}) : I(\text{Paper}) \rightarrow \text{Set}(I(\text{Person}))$

$I(\text{paper}) : I(\text{Person}) \rightarrow \text{Seq}(I(\text{Paper}))$       **(default role name)**

# Operations

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## Semantics

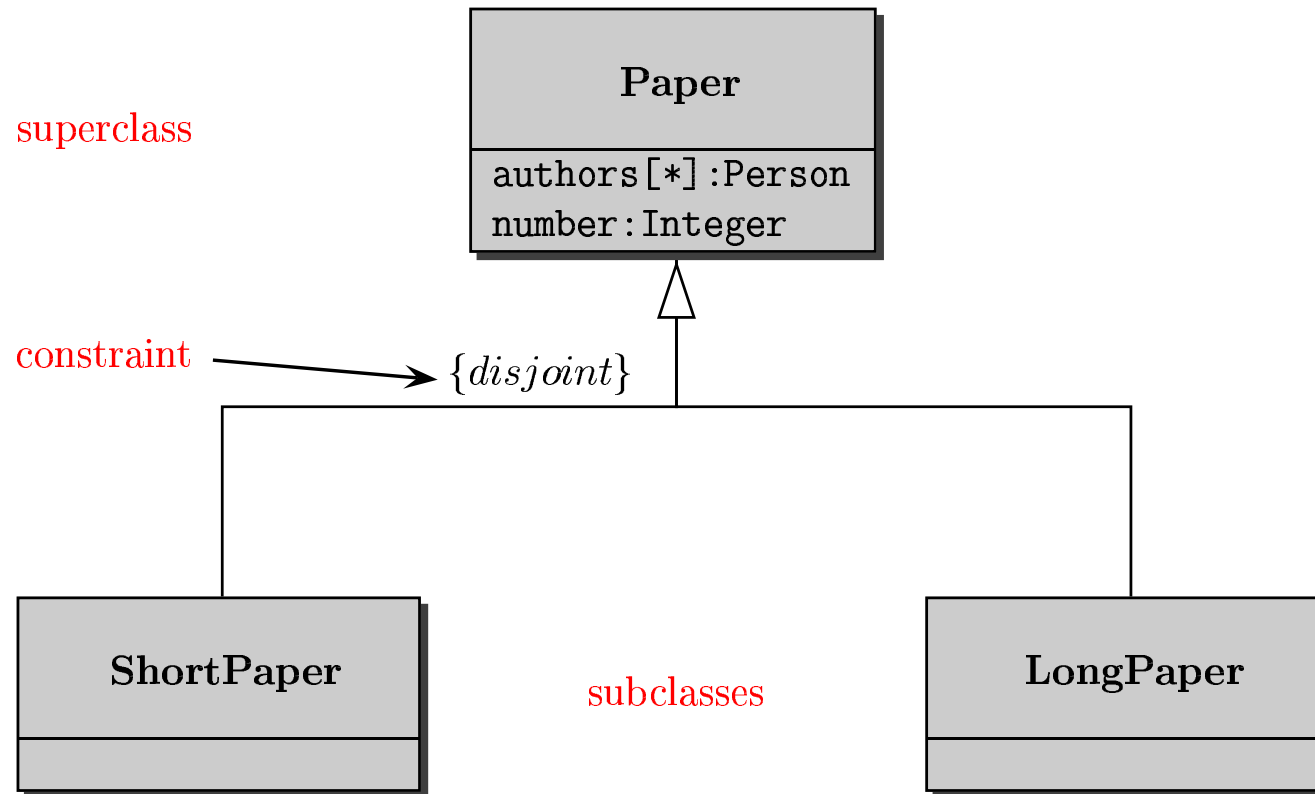
Transition from snapshot to snapshot  
(relation between sets of snapshots)

**Semantics of queries (operations without side-effects)**

**Partial functions**

# Subclasses

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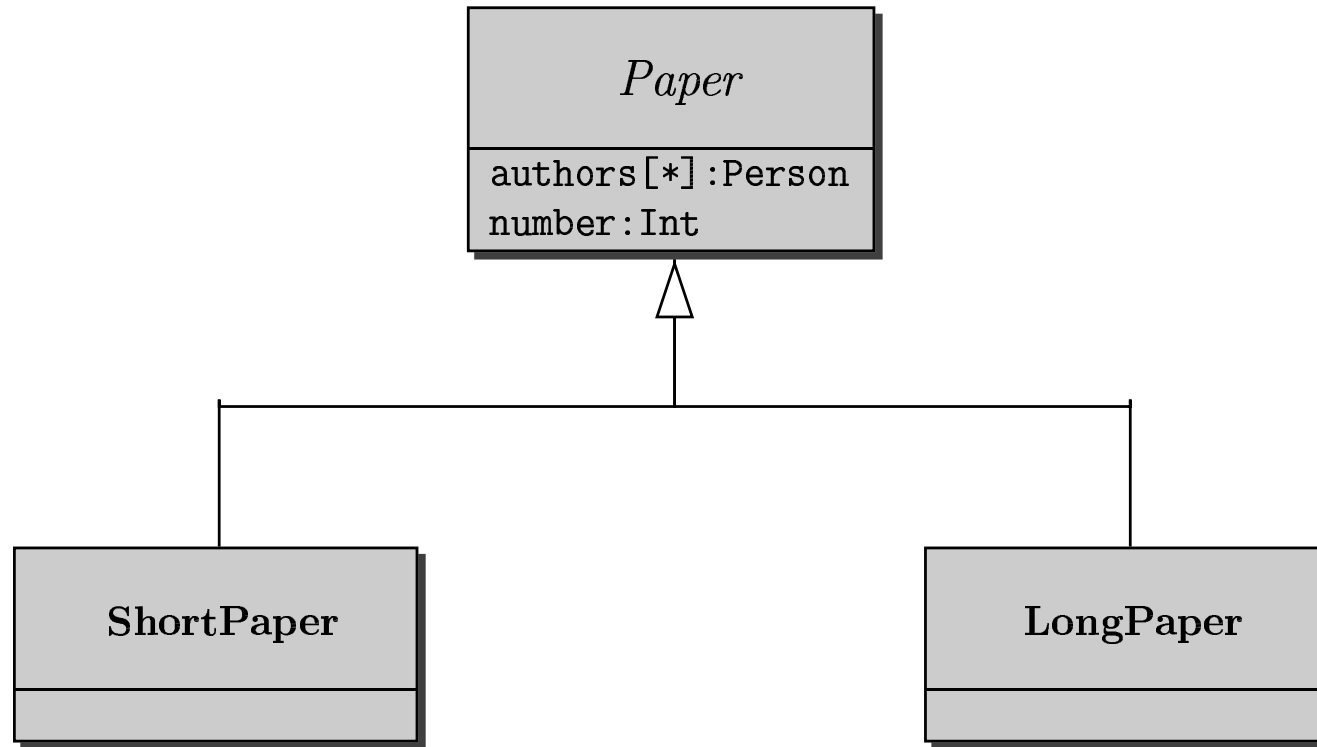
## Semantics

**subclass relation:**  $I(\text{ShortPaper}) \subset I(\text{Paper})$

**constraint:**  $I(\text{ShortPaper}) \cap I(\text{LongPaper}) = \emptyset$

# Abstract Classes

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## Semantics

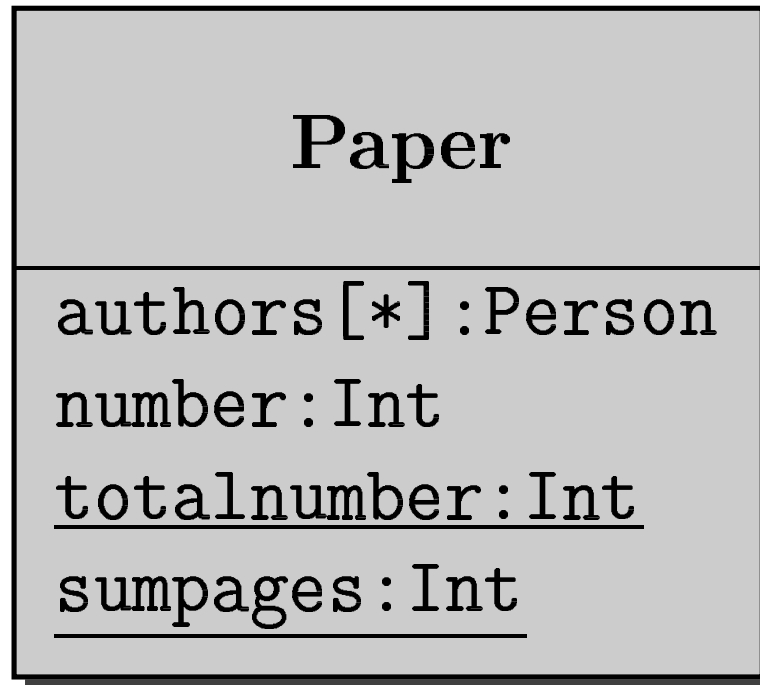
$$I(\textit{Paper}) = I(\textit{ShortPaper}) \cup I(\textit{LongPaper})$$

**No “direct” elements in  $I(\textit{Paper})$**



# Class Attributes

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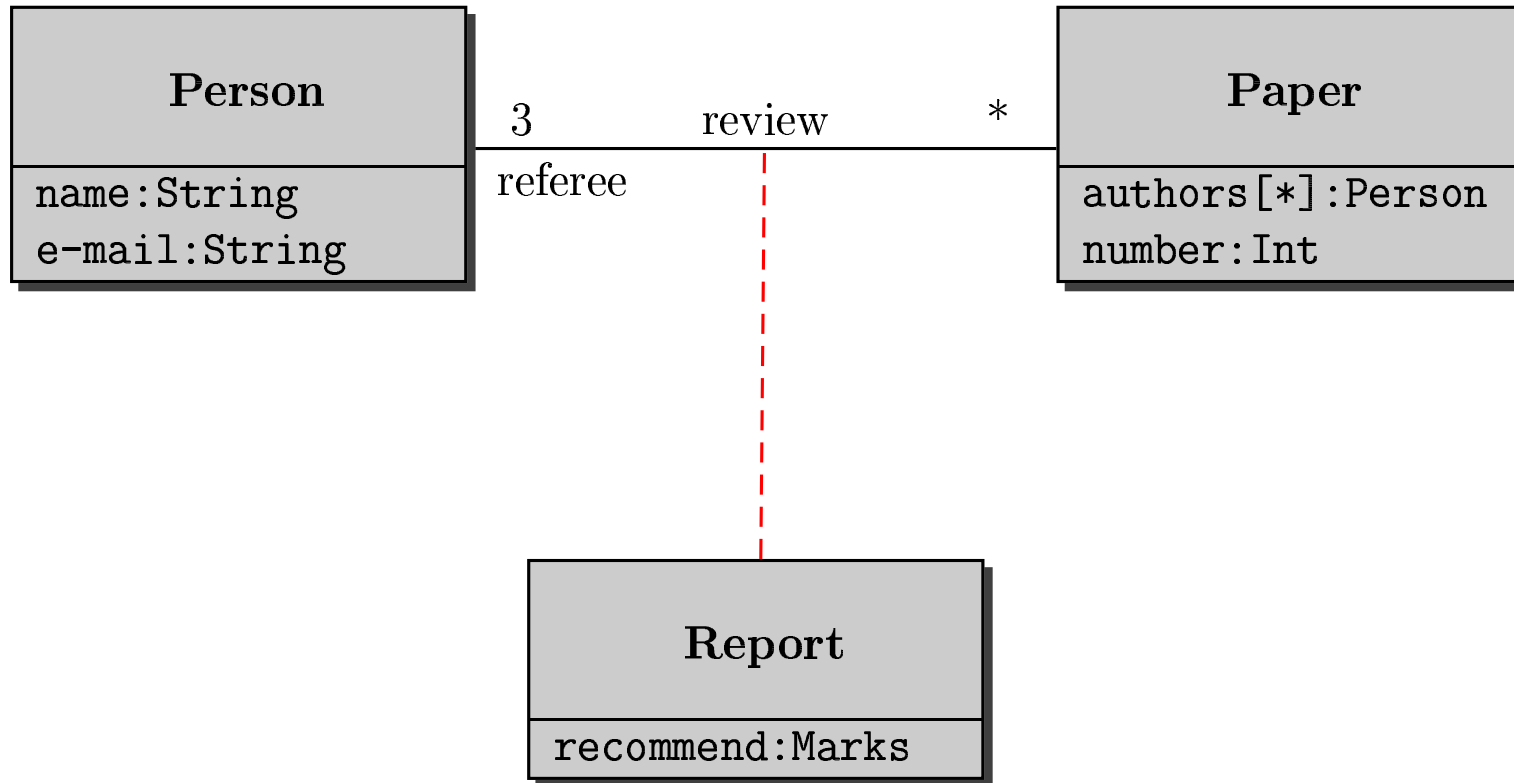
## Semantics

$I(\text{totalnumber})$  is an element of  $I(\text{Int})$

(i.e., `Paper.totalnumber` is a constant)

# Association Class

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## Semantics

$I(\text{Report})$  is a subset of  $I(\text{Person}) \times I(\text{Paper})$

# Data Types

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«data type»  
Integer

```
=(i2:Integer):Boolean
+(i2:Integer):Integer
+(i2:Real):Real
-(i2:Integer):Integer
-(i2:Real):Real
(i2:Integer):Integer
(i2:Real):Real
\ (i2:Integer):Real
\ (i2:Real):Real
abs:Integer
div(i2:Integer):Integer
mod(i2:Integer):Integer
max(i2:Integer):Integer
min(i2:Integer):Integer
```

«data type»  
String

```
=(i2:String):Boolean
size:Integer
concat(string2:String):String
toUpper(string2:String):String
toLowerCase(string2:String):String
substring(lower:Integer, upper:Integer):String
```

# Data Types

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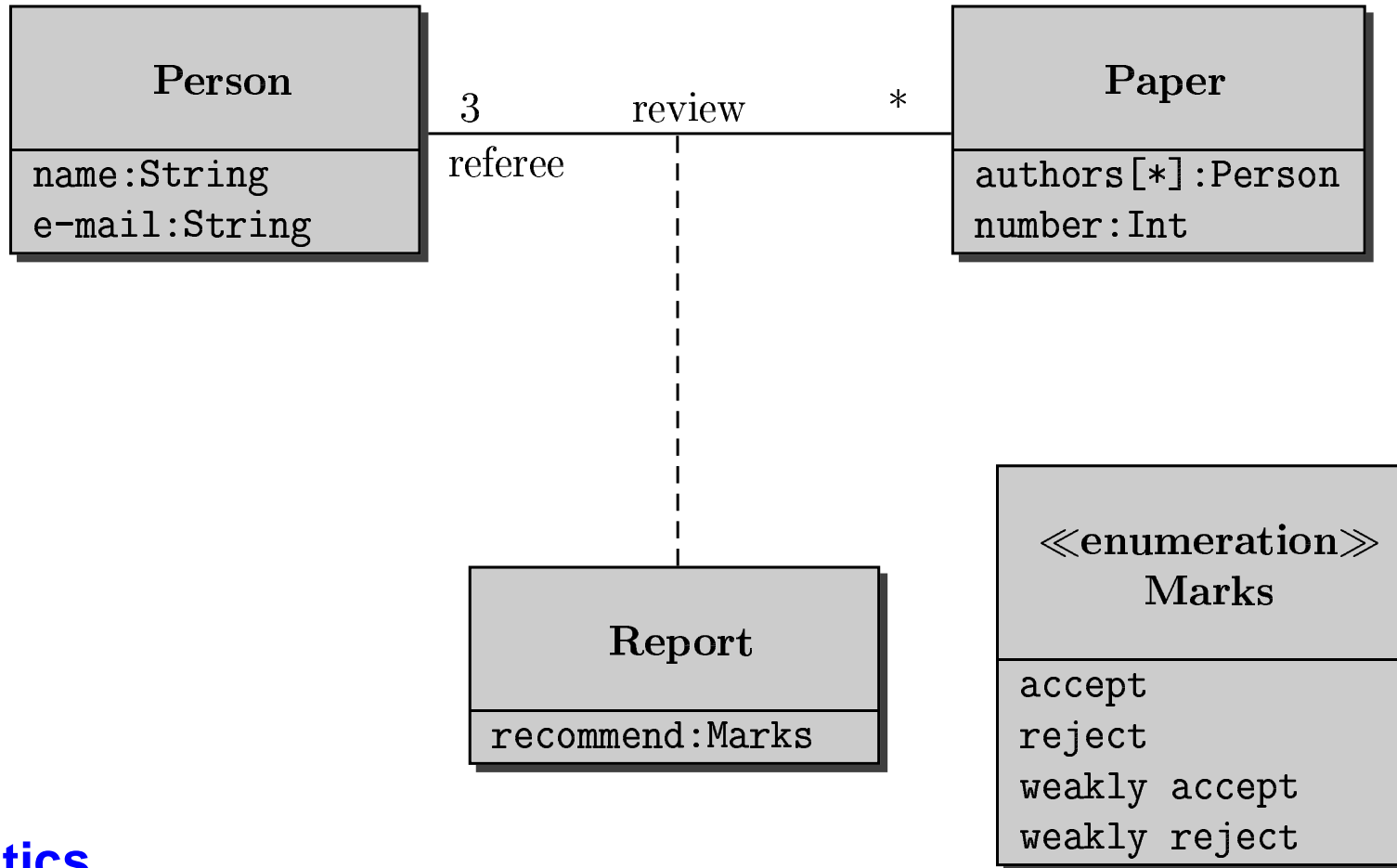
## Semantics

*I*(Integer) is the same in all snapshots

All operations are queries (no side effects)

No attributes

# Enumerations



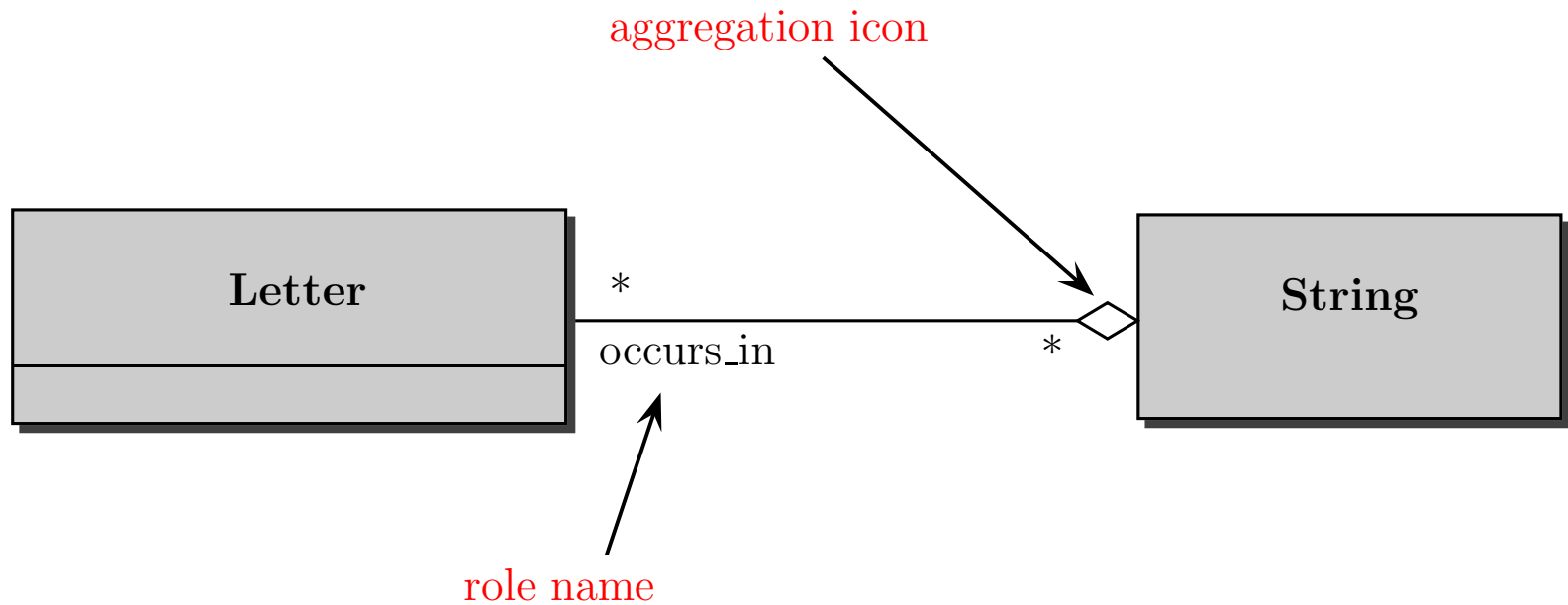
## Semantics

### Special kind of data type

$$I(\text{Marks}) = \{\text{accept}, \text{reject}, \text{weakly accept}, \text{weakly reject}\}$$

# Aggregations

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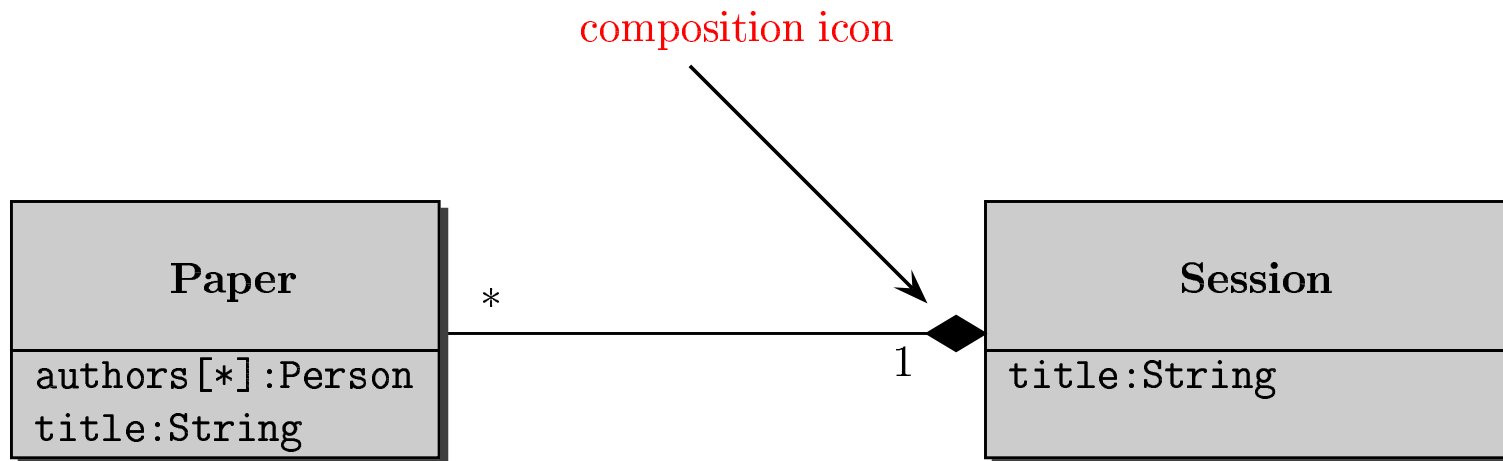


## Semantics

Same (formal) semantics as an associations

# Compositions

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## Semantics

Same (formal) semantics as an associations

# Example: The Composite Pattern

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