
Formal Verification of Software

Dynamic Logic for Java

Bernhard Beckert



UNIVERSITÄT KOBLENZ-LANDAU

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KeY Supports Java Card as Target Language

What is Java Card?

- **Subset of Java**
- **Sun's official standard for SMARTCARDS and embedded devices**

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- **multi-threading**
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Application Area

- **security critical**
- **financial risk**
(e.g. exchanging smart cards
is expensive)

Academic vs. Real-world Languages

Problems to address

Pointers / objects attributes

Modelled as non-rigid constants and functions

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if ((y=3) + y < 0) .. else ..
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Aliasing

Different names may refer to the same location, for example

$o.a, u.a$ in a state g where $g \models o \doteq u$

Other Issues (Later)

Further supported Java Card features

- ▶ method invocation, dynamic binding

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- ▶ **finiteness of integer data types**

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- ▶ **object creation and initialisation**
- ▶ **arrays**
- ▶ **finiteness of integer data types**
- ▶ **transactions**

Handling Object Attributes

Similar concepts

- **Object attributes**
- **Arrays**
- **Pointers**

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Non-rigid functions

Attributes are considered to be **non-rigid functions** on objects

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Extended to program variables

Program variables are considered to be **non-rigid constants**

Side Effects: Symbolic Execution Paradigm

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Solution:

- Calculus rules realise a stepwise symbolic execution of the programs (program transformation)
- Restrict applicability of some rules. For example, `if-then-else` is only applicable, if the guard is free of side-effects

Rule Application for if-then-else

$$\Gamma \vdash \langle \text{if } ((y = 3) + y < 0) \{ \alpha \} \text{ else } \{ \beta \} \rangle \Phi, \Delta$$

Rule Application for if-then-else

$$\frac{\Gamma \vdash \langle \text{boolean guard} = (y = 3) + y < 0; \text{if (guard)}\{\alpha\} \text{else}\{\beta\} \rangle \Phi, \Delta}{\Gamma \vdash \langle \text{if } ((y = 3) + y < 0)\{\alpha\} \text{else}\{\beta\} \rangle \Phi, \Delta}$$

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Assignment in the Classical Version

Classical rule for assignment

$$\frac{\Gamma^{x \leftarrow y}, x \doteq t^{x \leftarrow y} \quad \vdash \quad \Phi, \Delta^{x \leftarrow y}}{\Gamma \quad \vdash \quad \langle x = t \rangle \Phi, \Delta} \text{ (} y \text{ new variable)}$$

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Problems:

- *cannot* be handled as substitution

- aliasing:
$$\frac{?}{o.a \doteq 3 \quad \vdash \quad \langle u.a = 5; \rangle \phi}$$

Requires to split the proof for the cases $o = u$ and $o \neq u$.

The Active Statement in a Program

Example

$$\underbrace{l:\{\text{try}\{ i=0; j=0; \} \text{ finally}\{ k=0; \}\}}_{\pi}$$

ω

first active command $i=0;$

non-active prefix π

rest ω

Updates: Delayed Substitutions

Syntax: Updates are syntactical elements

$$\{loc := val\}\Phi \textbf{ or } \{loc := val\}t$$

where

loc **either a**

- **program variable** x
- **an attribute** $o.attr$ **or**
- **an array access** $a[i]$

val **a logical term (no side effects)**

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val **a logical term (no side effects)**

Semantic:

$$g \models \{loc := val\}\Phi \quad \textbf{iff} \quad g' \models \Phi \textbf{ where } g' = g_{loc}^{val}$$

Assignment Rule in KeY

$$\frac{\Gamma \vdash \{\text{loc} := \text{val}\} \langle \pi \ \omega \rangle \Phi, \Delta}{\Gamma \vdash \langle \pi \ \text{loc} = \text{val}; \ \omega \rangle \Phi, \Delta}, \text{ where } \text{loc}, \text{ val} \text{ side effect free}$$

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Advantages:

- no renaming as in the classical version
- delayed proof branching

$\Gamma \vdash \langle x = 3; \ x = 4; \rangle \Phi$ **or**

$\Gamma \vdash \langle o.a = 3; \ o.a = 4; \rangle \Phi$

Conditional Terms

Use conditional terms to delay splitting further

$$(s[t_1 \text{ ? } = t_2] \mapsto e)^{I,\beta} = \begin{cases} e^{I,\beta} & t_1^{I,\beta} = t_2^{I,\beta} \\ (s[t_1])^{I,\beta} & \textbf{otherwise} \end{cases}$$

Application of updates \mathcal{U}

**Application on
program variable**

$$\{x := t\} y \quad \rightsquigarrow \quad y$$

$$\{x := t\} x \quad \rightsquigarrow \quad t$$

$$\{o.a := t\} y \quad \rightsquigarrow \quad y$$

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Application on attribute

$$\{o.a := t\} o.a \quad \rightsquigarrow \quad t$$

$$\{o.a := t\} u.a \quad \rightsquigarrow \quad (\{o.a := t\} u? = o).a \mapsto t$$

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Application stops before modal operators, e.g.

$$\{o.a := t\} \langle \alpha \rangle \Phi \rightsquigarrow \{o.a := t\} \langle \alpha \rangle \Phi$$

Application is shoved over operators to the subformulas (terms)

$$\{o.a := t\} \Phi \wedge \Psi \rightsquigarrow \{o.a := t\} \Phi \wedge \{o.a := t\} \Psi$$

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Example

$$\{o.a := o\} o.a.a.b$$

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Example

$$\{o.a := o\} o.a.a.b \rightsquigarrow \{o.\textcolor{blue}{a} := o\} o.a.a.\textcolor{red}{b}$$

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Parallel Updates

Computing update followed by update

$$\{l_1 := r_1\} \{l_2 := r_2\} = \{\{l_1 := r_1\}, \{\{l_1 := r_1\} \downarrow l_2 := \{l_1 := r_1\} r_2\}\}$$

where $u \downarrow l = \begin{cases} x & \text{if } l = x \text{ is a program variable} \\ (u \ u).a & \text{if } l = u.a \end{cases}$

Results in parallel update: $\{l_1 := v_1, \dots, l_n := v_n\}$

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Semantics

- All l_i and v_i computed in old state
- All updates done simultaneously
- If conflict $l_i = l_j, v_i \neq v_j$ later update wins

Quantifying over Program Variables

Cannot quantify over program variables (non-rigid constants)

Non allowed: $\forall i:\text{int} (\langle \alpha(i) \rangle F)$

Non allowed: $\forall n (\langle \alpha(n) \rangle F)$

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Solution

$\forall n \{i := n\} \langle \alpha(i) \rangle F$

Abrupt Changes of the Control Flow

Abrupt Termination: Redirection of the control flow by

`return, break, continue` or Exceptions

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⟨try{  
    a = a/b;  
    a = a + 1;  
} catch(Exception e) {...}  
finally {...}⟩ Φ
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Decomposition Rule
not applicable

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Decomposition Rule
not applicable

Solution: The rules work on the first active statement

$$\frac{\Gamma \vdash \langle \pi \text{ stmt}' ; \omega \rangle \Phi, \Delta}{\Gamma \vdash \langle \pi \text{ stmt} ; \omega \rangle \Phi, \Delta}$$

Catch Throw Exception

Rule

$$\Gamma \vdash \langle \text{try}\{ \text{throw exc}; p \}$$
$$\text{catch (Exception } e) \{q\}$$
$$\text{finally}\{r\} \rangle \Phi, \Delta$$

Catch Throw Exception

Rule

$$\frac{\Gamma \vdash \langle \text{if } (\text{exc } \text{instanceof } \text{Exception}) \{ \\ \quad \text{try}\{\text{e} = \text{exc}; \text{q}\} \text{finally}\{\text{r}\} \\ \} \text{ else } \{ \text{r throw exc; } \} \rangle \Phi, \Delta}{\Gamma \vdash \langle \text{try}\{ \text{throw exc; } \text{p} \} \\ \quad \text{catch } (\text{Exception } \text{e}) \{ \text{q} \} \\ \quad \text{finally}\{\text{r}\} \rangle \Phi, \Delta}$$