

# Tutorial

## Integrating Object-oriented Design and Deductive Verification of Software

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Part I

## Introduction

## What is this Tutorial about?

- Design
- Formal specification
- Deductive verification

of

- Object-oriented software



This tutorial has been developed in the KeY project.  
The demos will use the KeY tool.

## KeY Project Partners



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[www.key-project.org](http://www.key-project.org)

## Integrated Formal Methods

### Specification

- UML + Object Constraint Language (OCL)
- Java Modeling Language (JML)

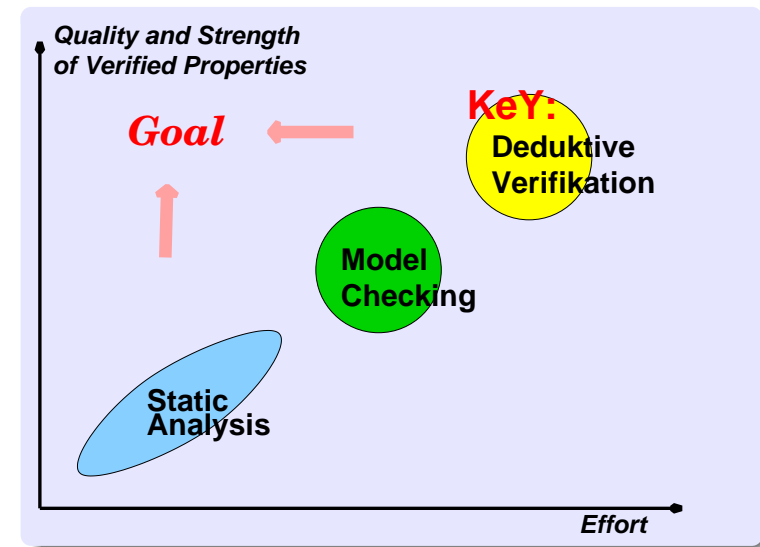
### Verification

- Dynamic Logic
- Decision procedures

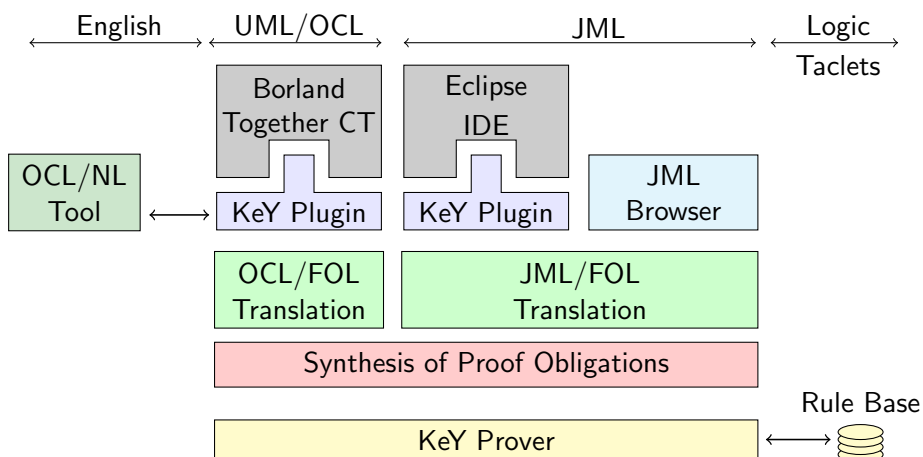
### And ...

- Static analysis
- Test case generation

## Different Approaches



## Architecture of the KeY Tool

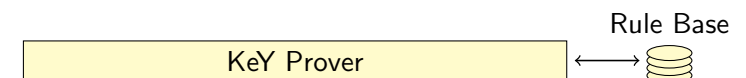


## Choices for the Rule Base

In this tutorial:  
100% Java Card

Other rule bases:

- ODL, a minimal abstract object oriented language
- A subset of the C language
- ASM, Abstract State Machines [Stanislas Nachen, ETH Zürich]
- HyKeY, differential dynamic logic for hybrid systems [André Platzer, Univ. of Oldenburg]



# Java Card

## What is Java Card?

- Subset of Java, but with transaction concept
- Sun's official standard for SMART CARDS and embedded devices

## Why Java Card?

Good example for real-world object-oriented language

### Java Card has *no*

- garbage collection
- dynamical class loading
- multi-threading
- floating-point arithmetic

### Application areas

- security critical
- financial risk  
(e.g. exchanging smart cards is expensive)



## Part II

### Specification

#### 3 Design by Contract

#### 4 OCL Specification

#### 5 JML Specification



## Part II

### Specification



### Design by Contract

#### Class

Invariant

#### Operation

Precondition

Modifies Clauses

Postcondition

Termination, more precisely: normal or exceptional



## Part II

### Specification

3 Design by Contract

4 OCL Specification

5 JML Specification

### Design by Contract with OCL

Class

Invariant

Operation

Precondition

Postcondition

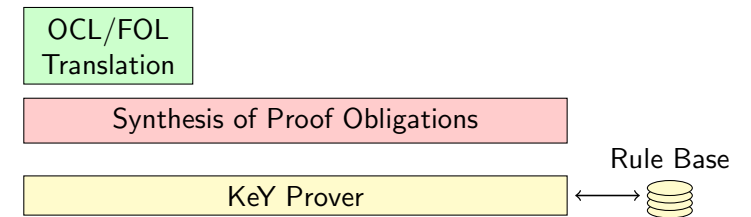
Modifies Clauses

Termination

## OCL: Object Constraint Language

### Object Constraint Language

- Part of the OMG standard UML
- Present Version: 2.0
- Adds formal constraints to UML (class) diagrams
- Accessible to people without a strong mathematical background



### Design by Contract with OCL

```
context ATM
```

```
inv: 0 <= self.wrongPinCounter and  
self.wrongPinCounter <= 2
```

```
context ATM::enterPin(pin: Integer)
```

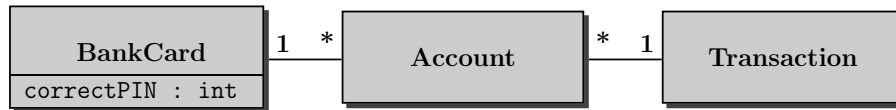
```
pre: insertedCard <> null and not customerAuthenticated  
and not pin = insertedCard.correctPIN  
and wrongPINCounter < 2
```

```
post: wrongPINCounter = wrongPINCounter@pre + 1  
and not customerAuthenticated
```

Modifies Clauses not explicitly supported by OCL

Termination specification not explicitly supported by OCL

## OCL Constraints on the UML Class Diagram Level



## Proof Obligations

```
context C
inv: I

context D extends C
inv: J
```

### Behavioural Subtyping for classes

For all instances  $o$  of  $D$  :  $o.J$  implies  $o.I$ .

## Proof Obligations

```
context C::op1
pre: pre1
post: post1

context D::op1
pre: pre2
post: post2
```

$D$  extends  $C$

### Behavioural Subtyping for operations

$pre1$  implies  $pre2$  and  
 $post2$  implies  $post1$

## Proof Obligations

```
context C::op
pre: pre
post: post
```

Implementation  $p$  of  $op$ .

### Ensures Postcondition

If  $p$  is started in a state satisfying  $pre$   
then  $p$  terminates and  
in the final state  $post$  is true.

## Proof Obligations

```
context C::op
pre: pre
post: post
```

```
context C
inv: I
```

Implementation  $p$  of  $op$ .

### Preserves Invariant

If  $p$  is started in a state satisfying  $pre$  and  $I$   
then  $p$  terminates and in the final state  $I$  is again true.

## Part II

## Specification

3 Design by Contract

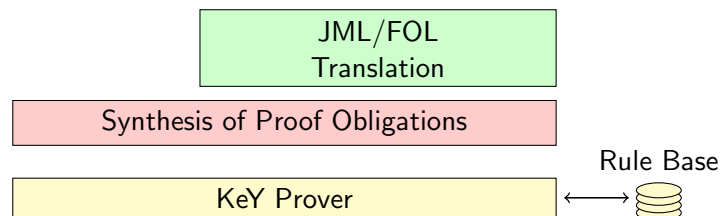
4 OCL Specification

5 JML Specification

## JML: Java Modeling Language

### Java Modeling Language

- Behavioral interface specification language for Java
- International community effort
- More and more tools:  
Runtime checkers, static analysis, program verification



## Design by Contract with JML (Invariants)

```
public class ATM {

    /*@ private invariant wrongPINCounter >= 0 &&
       wrongPINCounter <= 2
    @*/

    private BankCard insertedCard = null;
    private boolean customerAuthenticated = false;
    private int wrongPINCounter = 0;

    public void enterPIN (int pin) { ...
    }
}
```

## Design by Contract with JML (Operation Contracts)

```
public class ATM {  
  
  /*@ public normal_behavior  
   @ requires   insertedCard != null;  
   @ requires   !customerAuthenticated;  
   @ requires   pin != insertedCard.correctPIN;  
   @ requires   wrongPINCounter < 2;  
   @ ensures    wrongPINCounter ==  
                \old(wrongPINCounter) + 1;  
   @ assignable wrongPINCounter;  
   @  
   @ also ...  
  */  
  public void enterPIN (int pin) { ...  
  }  
}
```

## Another Example

```
public class Test {  
  private int idx;  
  
  /*@ requires precondition @ */  
  /*@ ensures postcondition @ */  
  void swapMax(int[] a) {  
    int counter = -1; idx = 0;  
  
    /*@ loop_invariant @*/  
    while (++counter < a.length) {  
      if (a[counter] > a[idx]) idx=counter;  
    }  
    int tmp=a[idx]; a[idx]=a[0]; a[0]=tmp;  
  }  
}
```

## JML Specification of swapMax

```
/*@ requires a!=null && a.length > 0;  
  @ ensures  
  @   (\forall int x; x==idx;  
  @   \old(a[0])==a[x] && \old(a[x])==a[0]) &&  
  @   (\forall int i; 0 <= i && i<\old(a.length);  
  @   a[0] >= a[i] &&  
  @   (i!=0 && i!=idx ==> a[i]==\old(a[i])));  
  @ diverges false;  
  @ */  
  void swapMax(int[] a) { ... }
```

## JML Loop Invariant

```
/*@ loop_invariant  
  @   -1<=counter && counter<=a.length &&  
  @   0<=idx && idx<a.length &&  
  @   (\forall int x; x>=0 && x<=counter;  
  @   a[idx]>=a[x]);  
  @ decreases (a.length - counter);  
  @ */  
  
  while (++counter < a.length) {  
    if (a[counter] > a[idx])  
      idx=counter;}
```

## Proving Postconditions for *swapMax*

After termination of the loop, we have ...

```
\forall i int; ((0 <= i & i <= a.length) ->
           a[idx] >= a[i])
```

It is also easy to show that ...

```
tmp = a[idx]; a[idx] = a[0]; a[0] = tmp;
```

has as post-condition

```
\forall i int; ((0 <= i & i <= a.length & i ≠ 0 & i ≠ idx) ->
           a[i] = \olda[i])
```

But ...

Loop invariant needs to be strengthened!

**SECOND DEMO**

## Improved JML Loop Invariant

```
/*@ loop_invariant
   @   -1<=counter && counter<=a.length &&
   @   0<=idx      && idx<a.length      &&
   @   (\forall int x; x>=0 && x<=counter;
   @       a[idx]>=a[x]);
   @ decreases (a.length - counter);

   @ assignable idx, counter;

   @*/

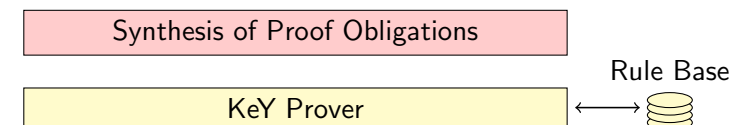
while (++counter<a.length) {
    if (a[counter] > a[idx])
        idx=counter;}

```

## Proof Obligations

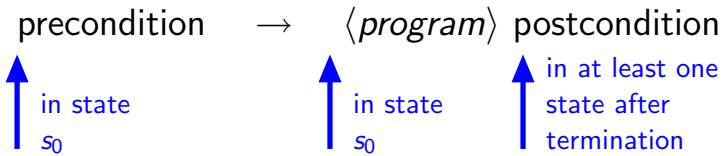
### Proof Obligations

- Behavioural Subtyping for classes
- Behavioural Subtyping for operations
- Strong Operation Contract
- Ensures Postcondition
- Preservation of Invariants
- Correctness of Modifies Clauses

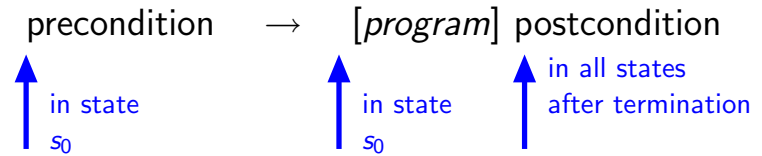




## Total Correctness Statement



## Partial Correctness Statement

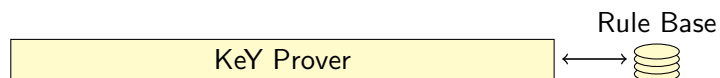


## Specification in Dynamic Logic

```

\programVariables {int pin; ATM self; int _pin; ...}
\problem {
  \forall ATM x0;
    x0.wrongPINCounter = ATM::wrongPINCounter@pre(x0) &
    !self.insertedCard = null &
    !self.customerAuthenticated = TRUE &
    !pin = self.insertedCard.correctPIN &
    self.wrongPINCounter < 2
  ->
  \< self.enterPIN(_pin)@ATM;\> self.wrongPINCounter =
    ATM::wrongPINCounter@pre(self) + 1
}

```



## Part III

## Logic and Calculus

## Logic and Calculus

- 6 Java Card DL
- 7 Sequent Calculus
- 8 Rules for Programs: Symbolic Execution
- 9 A Calculus for 100% Java Card
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## Why Dynamic Logic?

- Transparency wrt target programming language
- More expressive and flexible than Hoare logic
- Can use reference implementations instead of first-order theories
- Symbolic execution is a natural **interactive** proof paradigm
- Proven technology that scales up

## Syntax and Semantics

### Syntax

- Basis: Typed first-order predicate logic
- Modal operators  $\langle p \rangle$  and  $[p]$  for each (Java Card) program  $p$
- Class definitions in background (not shown in formulas)

### Semantics

- Operators refer to the final state of  $p$
- $[p] F$ : If  $p$  terminates, then  $F$  holds in the final state  
(partial correctness)
- $\langle p \rangle F$ :  $p$  terminates and  $F$  holds in the final state  
(total correctness)

Java Card DL formulas contain unaltered Java Card source code

## First-Order Formula Syntax

ASCII syntax, keywords preceded by '\'

### Logical operators

& and  
 | or  
 → implication  
 ↔ equivalence  
 ! negation

### Logical constants

true  
 false

### Conditional terms

\if(...)\then(...)\else(...)

### Quantifiers

\forall  
 \exists

## Dynamic Logic Example Formulas

$(\text{balance} > 1 \ \& \ \text{amount} > 1) \rightarrow \langle \text{charge}(\text{amount}); \rangle (\text{balance} > 1)$

$\langle x = 1; \rangle ([\text{while}(\text{true}) \{\}] \text{false})$

Syntax? ok

- Program formulas can appear nested



## Rigid and Flexible Terms

### Example

$\langle \text{int } i; \rangle \backslash \text{forall } \text{int } x; (i + 1 = x \rightarrow \langle i++; \rangle (i = x))$

- Interpretation of  $i$  depends on computation state  $\Rightarrow$  flexible
- Interpretation of  $x$  and  $+$  **must not** depend on state  $\Rightarrow$  rigid

Locations are always flexible  
Logical variables, standard functions are always rigid



## Variables

- Logical variables disjoint from program variables
  - No quantification over program variables
  - Programs do not contain logical variables
  - "Program variables" actually non-rigid functions

$\backslash \text{exists } \text{int } x; ([x = 1; ] (x = 1))$

Syntax? bad

- $x$  cannot be a **logical variable**, because it occurs in the program
- $x$  cannot be a **program variable**, because it is quantified

$\langle \text{int } x; \rangle \backslash \text{forall } \text{int } \text{val}; ((\langle p \rangle x = \text{val}) \leftrightarrow (\langle q \rangle x = \text{val}))$  Syntax? ok

- $p, q$  equivalent relative to computation state restricted to  $x$



## Type System

### Static types

- Partially ordered finite type hierarchy
- Terms are statically typed (like Java expressions)
- Type casts in logic

### Dynamic types

- Each term value has a dynamic type
- Dynamic type depends on state
- Dynamic types conform to static types
- Type predicates in logic



## Kripke semantics

- Semantics of a Java program is a partial function from states to states
- $\langle p \rangle F$  true in state  $s$  iff
  - $p$  terminates and  $F$  holds in the final state  $s'$  that is reached from  $s$  by running  $p$
- A Java Card DL formula is valid iff it is true in all states

We need a calculus for checking validity of formulae

# Sequents and their Semantics

## Syntax

$$\underbrace{\psi_1, \dots, \psi_m}_{\text{Antecedent}} \implies \underbrace{\phi_1, \dots, \phi_n}_{\text{Succedent}}$$

where the  $\phi_i, \psi_i$  are formulae (without free variables)

## Semantics

Same as the **formula**

$$(\psi_1 \& \dots \& \psi_m) \rightarrow (\phi_1 \mid \dots \mid \phi_n)$$

# Logic and Calculus

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# Sequent Rules

## General form

$$\text{RULE NAME} \frac{\overbrace{\Gamma_1 \implies \Delta_1 \quad \dots \quad \Gamma_r \implies \Delta_r}^{\text{Premises}}}{\underbrace{\Gamma \implies \Delta}_{\text{Conclusion}}}$$

( $r = 0$  possible)

## Soundness

If all premisses are valid, then the conclusion is valid

## Some Simple Sequent Rules

$$\text{NOT\_LEFT} \frac{\Gamma \implies A, \Delta}{\Gamma, !A \implies \Delta}$$

$$\text{IMP\_LEFT} \frac{\Gamma \implies A, \Delta \quad \Gamma, B \implies \Delta}{\Gamma, A \rightarrow B \implies \Delta}$$

$$\text{CLOSE\_GOAL} \frac{}{\Gamma, A \implies A, \Delta} \quad \text{CLOSE\_BY\_TRUE} \frac{}{\Gamma \implies \text{true}, \Delta}$$

$$\text{ALL\_LEFT} \frac{\Gamma, \forall x; \phi, \{x/e\}\phi \implies \Delta}{\Gamma, \forall x; \phi \implies \Delta}$$

where  $e$  var-free term of type  $t' \prec t$

## Part III

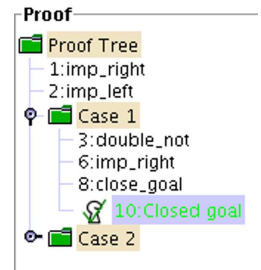
### Logic and Calculus

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## Sequent Calculus Proofs

### Proof tree

- Proof is tree structure with goal sequent as root
- Rules are applied from conclusion (old goal) to premisses (new goals)
- Rule with no premiss closes proof branch
- Proof is finished when all goals are closed



## Proof by Symbolic Program Execution

- Sequent rules for program formulas?
- What corresponds to top-level connective in a program?

### The Active Statement in a Program

Example

$$l: \underbrace{\{\text{try}\{ i=0; j=0; \}}_{\pi} \text{ finally}\{ k=0; \}}$$

active statement  $i=0;$   
 non-active prefix  $\pi$   
 rest  $\omega$

## Proof by Symbolic Program Execution

- Sequent rules execute symbolically the first active statement
- Sequent proof corresponds to symbolic program execution

**Example: The rule for if-then-else (SIMPLIFIED VERSION!)**

$$\frac{\Gamma, B ==> \langle \pi \ p \ \omega \rangle \phi, \Delta \quad \Gamma, !B ==> \langle \pi \ q \ \omega \rangle \phi, \Delta}{\Gamma ==> \langle \pi \ \text{if } (B) \{ p \} \ \text{else } \{ q \} \ \omega \rangle \phi, \Delta}$$

## Problems to Address

### Object attributes & arrays

Modelled as non-rigid functions

### Side effects

Expressions in programs can have side effects

#### Example

```
if ((y=3) + y < 0) {...} else {...}
```

### Aliasing

Different names may refer to the same location

#### Example

After `o.a=17;`, what is `u.a`?

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## Other Issues

### Further supported Java Card features

- method invocation with polymorphism/dynamic binding
- arrays
- abrupt termination
- throwing of `NullPointerException`s, etc.
- object creation and initialisation
- bounded integer data types
- transactions

All Java Card language features are fully addressed in KeY

## Java—A Language of Many Features

### Ways to deal

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose constructs in program logic

**Pro:** Feature needs not be handled in calculus

**Contra:** Modified source code

**Example in KeY:** Very rare: treating inner classes



## Java—A Language of Many Features

### Ways to deal

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose constructs in program logic

**Pro:** Flexible, easy to implement, usable

**Contra:** Not expressive enough for all features

**Example in KeY:** Complex expression eval, method inlining, etc., etc.



## Java—A Language of Many Features

### Ways to deal

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose constructs in program logic

**Pro:** No logic extensions required, enough to express most features

**Contra:** Creates difficult first-order POs, unreadable antecedents

**Example in KeY:** Dynamic types and branch predicates



## Java—A Language of Many Features

### Ways to deal

- Program transformation, up-front
- Local program transformation, done by a rule on-the-fly
- Modeling with first-order formulas
- Special-purpose constructs in program logic

**Pro:** Arbitrarily expressive extensions possible

**Contra:** Increases complexity of all rules

**Example in KeY:** Method frames, updates



## Handling Side Effects

### Problem

- Expressions may have side effects
- Terms in logic have to be side effect free

### Example

$(y=3) + y < 0$

does not only evaluate to a boolean value, but also assigns a value to  $y$



## Handling Side Effects

### Solution

- Calculus rules realise a stepwise symbolic evaluation (simple transformations)
- Restrict applicability of some rules (e.g., if-then-else)

### Example

$\text{if } ((y=3) + y < 0) \{ \dots \} \text{ else } \{ \dots \}$

rewritten into

```
y      = 3;
int    val1 = y;
int    val0 = val1 + y;
boolean guard = (val0 < 0);
if (guard) { ... } else { ... }
```



## Handling Assignment: Explicit State Updates

### Problem

Because of aliasing,  
assignment cannot be handled as syntactic substitution

### Solution

State updates as explicit syntactic elements

### Syntax

$\{loc := val\}\phi$

where (roughly)

- $loc$  is a program variable  $x$ , an attribute access  $o.a$ , or an array access  $a[i]$
- $val$  is same as  $val$ , a literal, or a logical variable



## Assignment Rule in KeY

$$\frac{\Gamma \implies \{loc := val\} \langle \pi \ \omega \rangle \phi, \Delta}{\Gamma \implies \langle \pi \ loc=val; \ \omega \rangle \phi, \Delta}$$

### Advantages

- no renaming required
- delayed proof branching

### Update simplification in KeY

KeY system has powerful mechanism for simplifying and applying updates

- eager simplification (also: parallel updates)
- lazy application





## Handling Abrupt Termination

Example: try-throw

- Abrupt termination handled by “simple” program transformations
- Changing control flow = rearranging program parts

### Example

TRY-THROW (exc simple)

$$\frac{\Gamma \implies \left\langle \begin{array}{l} \pi \text{ if (exc instanceof T)} \\ \quad \{ \text{try } \{ e = \text{exc}; r \} \text{ finally } \{ s \} \} \\ \text{else } \{ s \text{ throw exc}; \omega \end{array} \right\rangle \phi}{\Gamma \implies \langle \pi \text{ try}\{\text{throw exc}; q\} \text{ catch}(T e)\{r\} \text{ finally}\{s\}; \omega \rangle \phi}$$



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## Components of the Calculus

- 1 Non-program rules
  - first-order rules
  - rules for data-types
  - rules for modalities
  - the induction rule
- 2 Rules for reducing/simplifying the program (symbolic execution)  
Replace the program by combination of
  - case distinctions (proof branches) and
  - sequences of updates
- 3 Rules for handling loops
  - rules using loop invariants
  - rules for handling loops by induction
- 4 Rules for replacing a method invocations by the method's contract
- 5 Update simplification



## Taclets

### Taclets are the “rules” of the KeY system

Taclets...

- have logical content like rules of the calculus
- have pragmatic information for interactive application
- have pragmatic information for automated application
- keep all these concerns separate but close to each other
- can easily be added to the system
- are given in a textual format
- can be verified w.r.t. base taclets



## Taclet Syntax (by Example)

### Modus ponens: Rule

$$\frac{\Gamma, \phi, \psi \implies \Delta}{\Gamma, \phi, \phi \rightarrow \psi \implies \Delta}$$

### Modus ponens: Taclet

```
modus_ponens{
  \find (phi -> psi ==>)
  \assumes (phi ==>)
  \replacewith (psi ==>)
  \heuristics(simplify)
}
```



## Java Card Taclets

### Rule if\_else\_split

$$\frac{B = \text{TRUE} \implies \langle \pi \ p \ \omega \rangle F \quad B = \text{FALSE} \implies \langle \pi \ q \ \omega \rangle F}{\implies \langle \pi \ \text{if} \ (B) \ p \ \text{else} \ q \ \omega \rangle F}$$

where  $B$  is a Boolean expression without side effects

### Corresponding taclet

```
if_else_split {
  \find (==> <{.. if(#B) #p else #q ..}>post)
  \replacewith (==> <{.. #p ..}>post) \add (#B = TRUE ==>);
  \replacewith (==> <{.. #q ..}>post) \add (#B = FALSE ==>)
  \heuristics(if_split)
};
```



## An Axiom and a Branching Rule

### Closure rule

```
close_goal {
  \find (==> b)
  \assumes (b ==>)
  \closegoal
  \heuristics(closure)
};
```

### Cut rule

```
cut {
  \add (b ==>);
  \add (==> b)
};
```



## Taclets: Summary

### Taclets are ...

- simple and (sufficiently) powerful
- compact and clear notation
- no complicated meta-language
- easy to apply with a GUI
- validation possible



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## Validating Soundness of Proof Rules

### Bootstrapping

Validate a core set of rules,  
generate and prove verification conditions for additional rules

### Cross-verification

- against the BALI calculus for Java formalized in Isabelle/HOL  
[D. von Oheimb, T. Nipkow]
- against the Java semantics in the MAUDE system  
[J. Meseguer]

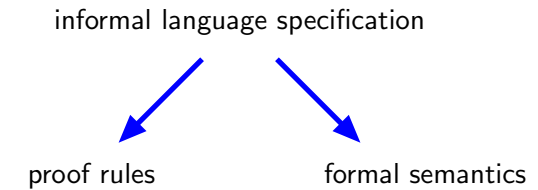
### Tests

Using the compiler test suite Jacks



## Verification Calculus Soundness

A fundamental problem!



## From the Java Language Specification

### PostIncrementExpression:

PostfixExpression ++

*At run time, if evaluation [...] completes abruptly, then the postfix increment expression completes abruptly and no incrementation occurs.*

*Otherwise, the value 1 is added to the value of the variable and the sum is stored back into the variable. Before the addition, binary numeric promotion is performed on the value [...]*

*The value of the postfix increment expression is the value of the variable before the new value is stored.*



## Rule for Postfix Increment

### Intuitive rule (not correct!)

$$\frac{==> \langle \pi \ x=y; \ y=y+1; \ \omega \rangle \phi}{==> \langle \pi \ x=y++; \ \omega \rangle \phi}$$

### But ...

$$x = 5 ==> \langle x=x++; \rangle (x = 6) \quad \text{INVALID}$$

### Correct rule

$$\frac{==> \langle \pi \ v=y; \ y=y+1; \ x=v; \ \omega \rangle \phi}{==> \langle \pi \ x=y++; \ \omega \rangle \phi}$$



## Part III

## Logic and Calculus

- 6 Java Card DL
- 7 Sequent Calculus
- 8 Rules for Programs: Symbolic Execution
- 9 A Calculus for 100% Java Card
- 10 Taclets and Taclet Language
- 11 Correctness of Proof Rules
- 12 Interactive and Automated Proof Construction



## From the Jacks Conformance Test Suite

```
class T1241r1a {
    final int i=1; static final int j=1;
    static { }
}

class T1241r1b {
    /*@ public normal_behavior
       @ ensures \result == 7; @ */
    public static int main() {
        int s = 0; T1241r1a a = null;
        s = s + a.j;
        try {s = s + a.i;}
        catch (Exception e) {
            s = s + 2; a = new T1241r1a();
            s = s + a.i + 3; }
        return s; }
}
```



## Interaction and Automation

For realistic programs:  
Fully-automated verification **impossible**



## Interaction and Automation

### Goal in KeY: Integrate automated and interactive proving

- All easy or obvious proof steps should be automated
- Sequents presented to user should be simplified as far as possible
- Primary steps that require interaction: induction, treatment of loops
- Taclets enable interactive rule application mostly using mouse

### Typical workflow when proving in KeY (and other interactive provers)

- 1 Prover runs automatically as far as possible
- 2 When prover stops user investigates situation and gives hints (makes some interactive steps)
- 3 Go to 1

## Extension of Proof: Application of Single Taclets

### Taclet application requires

- A proof goal
- Focus of rule application: term/formula in the goal
- Instantiation of schema variables

### Main procedure for applying a taclet interactively

- 1 Select an application focus using mouse pointer
- 2 Select a particular rule from the context menu
- 3 Instantiate schema variables

## Working with Sequents: Sequent View

### For goals (leaves of tree)

- Obtaining information about formulas/terms (press Alt key)
- Selecting formulas/terms, applying rules to them

```
Current Goal
  self_ATM_iv_0.accountProxies@(ATM)[i_j
    = i_jm1_iv3)
==>
  self_ATM_iv_0.insertedCard@(ATM).accountNumbe
  < 0,
  self_ATM_iv_0.online@(ATM) = TRUE,
  self_ATM_iv_0.insertedCard@(ATM).invalid@(Bank
  = TRUE,
  self_ATM_iv_0
  self_ATM_iv_0.commute_eq
  self_ATM_iv_0.rinse_noal
  self_ATM_iv_0.replacewith ( null = self_ATM_iv_0 ) TRUE
  self_ATM_iv_0.replace_known_right
  {b_4:=TRUE,
  hide_right
  pin:=pin_iv_0
  case_distinction
  self_ATM:=se
  \method=fn

Inner Node
  self_ATM_iv_0.centralHost@(ATM).accounts@(Centr
  = null,
  self_ATM_iv_0.insertedCard@(ATM).invalid@(Bank
  = TRUE,
  self_ATM_iv_0 = null,
  self_ATM_iv_0.accountProxies@(ATM) = null,
  self_ATM_iv_0.insertedCard@(ATM) = null,
  self_ATM_iv_0.customerAuthenticated@(ATM) = TRUE,
  self_ATM_iv_0.centralHost@(ATM) = null,
  \if (!self_ATM_iv_0.insertedCard@(ATM) = null)
  \then ({pin:=pin_iv_0,
  self_ATM:=self_ATM_iv_0}
```

## Applying Taclets using Drag-and-Drop

### Possible for taclets with find-part and one assumption, like ...

- Rewriting a term using an equation
- Instantiating formulas with universal-type quantifier

### Applying equations

- Drag the equation onto the term to be rewritten

```
Current Goal
a = b, c = b ==> a = c
```

### Instantiating quantifiers

- Drag instantiation term onto the quantified formula

```
Current Goal
p(x_0, v_0),
\forall s y; p(x_0, y)
==>
\exists s u; p(u, v_0)
```

## Means of Automation Implemented in KeY

- Parameterized strategies for applying rules automatically
- Free-variable first-order calculus (non-destructive, proof-confluent)
- Invocation of external theorem provers, decision procedures:
  - Simplify (from ESC/Java)
  - ICS
  - Any other with SMT-LIB interface

## Part IV

### Further Topics

## Strategies Currently Present in KeY

Strategies optimized for . . .

### Symbolic execution of programs

- Come in different flavours: with/without unwinding loops, etc.
- Concentrate on eliminating program and simplifying sequents

### Handling first-order logic

- Implements a complete first-order theorem prover
- Includes arithmetics solver

## Part IV

### Further Topics

#### 13 Dealing with Integers

#### 14 Proof Reuse

#### 15 Generating Test Cases

#### 16 Concurrency

## Specification of Integer Square Root

Taken from: Preliminary Design of JML [G. Leavens et al.]

```
/*@ requires y >= 0;
   @ ensures
   @ \result * \result <= y &&
   @ y < (abs(\result)+1) * (abs(\result)+1);
   @ */
public static int isqrt(int y)
```

But ...

$\backslash\text{result} = 1073741821 = \frac{\text{max\_int}-5}{2}$  satisfies spec for  $y = 1$ .

$1073741821 * 1073741821 = -2147483639 \leq 1$

$1073741822 * 1073741822 = 4 > 1$

## Examples

Valid for Java integers

- $\text{MAX\_INT} + 1 = \text{MIN\_INT}$
- $\text{MIN\_INT} * (-1) = \text{MIN\_INT}$
- $\exists x, y. (x \neq 0 \wedge y \neq 0 \wedge x * y = 0)$

Not valid for Java integers

- $\forall x. \exists y. y > x$

Not a sound rewrite rule for Java integers

- $x + 1 > y + 1 \rightsquigarrow x > y$

## Data Type Gap

Specification level: Abstract data types

- Integer ( $\mathbb{Z}$ )
- Set, List

Implementation level: Concrete programming language data types

- byte, short, int, long
- Array

## More Formal Semantics of Java Integer Types

Range of primitive integer types in Java

Type	Range	Bits
byte	$[-128, 127]$	8
short	$[-32768, 32767]$	16
int	$[-2147483648, 2147483647]$	32
long	$[-2^{63}, 2^{63} - 1]$	64

## Options for Integer Semantics Rules in KeY

### Java semantics

- Faithfully axiomatises the overflow semantics of Java integers
- Leads to hard verification problems (lack of intuition)

### Arithmetic semantics

- Leads to easier verification problems
- Incorrect

### Arithmetic semantics with overflow check

- Correct
- Leads to moderate verification problems
- Incomplete  
(there are programs that are correct despite overflows)

## Proof Reuse

### Basic Use Case

- 1 Verification attempt fails
- 2 Amend program
- 3 Recycle unaffected proof parts

### Example: Incremental Verification

- 1 Program correct w.r.t. arithmetic semantics? ✓
- 2 Program correct w.r.t. overflow checking semantics? ✗
- 3 Fix bug, reuse proof ✓

Successfully used in case studies

## Part IV

## Further Topics

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## Proof Reuse

### Observations

- Similar program rule applications focus on similar program parts
- Program rules applicable at a limited number of goals
- Proof structure follows program structure

### Steps

- 1 Identify changes in program (program diff)
- 2 Identify subproofs beginning with unaffected statements
- 3 Similarity-guided proof replay



## Further Topics

13 Dealing with Integers

14 Proof Reuse

15 **Generating Test Cases**

16 Concurrency

## Test Case Ingredients

### Generate unit tests

- Code fragment to be tested
- Test cases
- Test oracle
- Test setup for each execution path

## Generating Test Cases

### Testing makes sense, even in cases when a formal proof exists

- Testing can uncover bugs in environment (hardware, compiler, operating system, virtual machine)
- Testing can uncover specification errors
- Testing can uncover bugs w.r.t. unspecified properties (e.g. timing)
- Tests can be generated from incomplete proofs

### Idea: Use a formal proof to generate test cases

- KeY provides the path condition for each execution path
- High code coverage (feasible execution paths)
- For infinite number of paths:  
Unwind loops finite number of times, inline method bodies

## Example (Finite Number of Execution Paths)

### Compute the middle of three numbers

```
public static int middle(int x, int y, int z){
    int mid = z;
    if (y < z){
        if (x < y){
            mid = y;
        } else if (x < z){
            mid = x;
        }
    } else {
        if (x > y){
            mid = y;
        } else if (x > z){
            mid = x;
        }
    }
    return mid;
}
```

## Further Topics

13 Dealing with Integers

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## java.lang.StringBuffer

```

private char value [];
private int count;

public synchronized StringBuffer
    append(char c) {
    int newcount = count + 1;
    if (newcount > value.length)
        expandCapacity(newcount);
    value[count++] = c;
    return this;
}

```



## Verifying concurrent Java programs

Full reasoning about data

Beyond just safety or race detection

No abstractions



## Verify That...

$$\begin{aligned}
 &\text{strb}.\langle\text{lockcount}\rangle = 0 \wedge \neg\text{strb} = \text{null} \wedge \text{strb}.\text{count} = 0 \rightarrow \\
 &\quad \forall n. n > 0 \rightarrow \\
 &\quad \langle\{n\}\text{strb}.\text{append}(c);\{0\}\rangle \text{strb}.\text{count} = n \wedge \\
 &\quad \forall k. 0 \leq k < n \rightarrow \text{strb}.\text{value}[k] = c(p_1(k+1))
 \end{aligned}$$


## Three-Step Programme

- 1 Unfold
- 2 Prove atomicity invariant
- 3 Symbolic execution + induction

## Statistics

- Proof steps: 14622
- Branches: 238 (3 relevant)
- Interactions: 2
- Runtime: ~1 minute
- Result: conjecture false for  $n \geq MAX\_INT$

## Concurrency Verification Problems

- Number of threads
  - ↳ symmetry reduction (this work)
- Number of interference points
  - ↳ exploit locking, data confinement
- Java Memory Model
  - ↳ ?

## Alas...

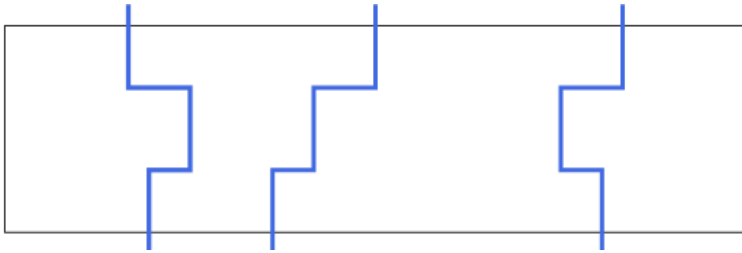
No thread identities **in** programs

No dynamic thread creation (but unbounded concurrency)

Currently only atomic loops

## The Calculus Is Built On...

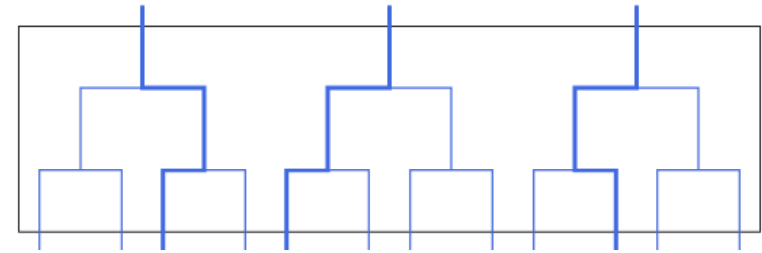
symmetry reduction



... and explicit scheduler formalization

## The Calculus Is Built On...

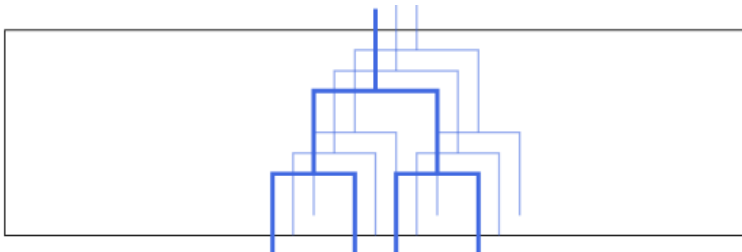
symmetry reduction



... and explicit scheduler formalization

## The Calculus Is Built On...

symmetry reduction



... and explicit scheduler formalization

Part V

**Wrap Up**

## Part V

### Wrap Up

#### 17 Case Studies

#### 18 Current Directions of Work

#### 19 Acknowledgments



## Algorithm Verification

### Schorr-Waite Algorithm

- Graph-marking algorithm (memory-efficient garbage collection)
- Very complicated loop invariant
- One single proof with 17,000 steps



## “Fundamental” Case Studies: Libraries

### Java Collections Framework (JCF)

- Part of JCF (treating sets) specified using UML/OCL
- Parts of reference implementation verified

### Java Card API Reference Implementation

- Covers whole of latest API used in practice (2.2.1)
- 60 classes, 4,500 lines of Java code
- Effort: 2–3 (expert) months



## Security Case Studies: Java Card Software

### Demoney

- Electronic purse application provided by Trusted Logic S.A.

### Mondex Card

- Smart card for electronic financial transactions
- Issued by NatWest in 1996
- Proposed as case study in Grand Challenge
- KeY used to verify a reference implementation in Java Card



## Safety Case Study

### Avionics Software

- Java implementation of a Flight Manager module at Thales Avionics
- Comprehensive specification using JML, emphasis on class invariants
- Verification of some nested method calls using contracts

### Virtual Machine for Real Time Security Java

- Verification of some library functions of the Jamaica VM from Aicas

## Some Current Directions of Research in KeY

- Multi-threaded Java
- Integration of deduction and static analysis
- Integration of verification and testing
- Counter examples
- Symbolic error propagation
- Verification of MISRA C
- Proof visualization, proving as debugging

Extension of dynamic logic for multi-threading  
Symbolic execution calculus  
Prototype available, `StringBuffer` class verified

## Part V

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Mutual call of analyser/prover, common semantic framework  
Implementation of static analysis in theorem proving frame

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Generation of test cases from proofs  
Symbolic testing  
New coverage criteria



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Generate counter example from failed proof attempt  
Counter example search as proof of uncorrectness



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Symbolic error classes modeled by formulas  
Error injection by instrumentation of Java Card DL rules  
Symbolic error propagation via symbolic execution



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## Part V

## Wrap Up

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## More Information

### The KeY Book

B. Beckert, R. Hähnle, P. H. Schmitt (eds.)

*Verification of Object-Oriented Software:  
The KeY Approach*

Springer-Verlag, LNCS 4334, 2007.



### Web site

[www.key-project.org](http://www.key-project.org)