Formal Property-Oriented Design of Voting Rules Using Composable Modules

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Abstract. Voting rules aggregate multiple individual preferences in order to make a collective decision. Commonly, these mechanisms are expected to respect a multitude of different notions of fairness and reliability, which must be carefully balanced to avoid inconsistencies.
We present an approach for the sound and flexible design of voting rules from composable modules. Formal composition rules guarantee social choice properties from properties of the individual components. The approach can be applied to many voting rules from the literature.

Keywords: Social choice · Formal correctness · Modular design.

1 Introduction

In an election, voters cast ballots to express individual preferences about eligible alternatives. From these preferences, a collective decision, i.e., a set of elected alternatives, is determined using a *voting rule*. Voting rules are commonly designed to meet various expectations for fairness and reliability, but no one general rule caters for every requirement, and every rule shows paradoxical behavior for some situation [1]. The *axiomatic method* permits the analysis of desired behavior by comparing and characterizing voting rules via rigorous guarantees in the form of formal properties. Designing voting rules towards such properties is generally challenging as their trade-off is inherently difficult and error-prone.
Contribution. We present an approach for the systematic and formal design of voting rules from compact composable modules with formal properties guaranteed by construction. This work gives the core component type and compositional structures, e.g., for sequential, parallel and loop composition, and illustrates how composition rules formally establish common social choice properties.

2 Property-Oriented Composition of Voting Rules

Electoral Modules. The foundation of our approach are *electoral modules*, a generalization of voting rules. Voting rules elect a set of alternatives from a profile, i.e., a sequence of ranked ballots, and a nonempty set of alternatives A. Electoral modules are more general as they do not need to make final decisions, but instead partition A into *elected*, *rejected* and *deferred* alternatives. Hence, if an electoral module always produces a nonempty set of elected alternatives $A_{elected}$, it directly induces a voting rule which elects $A_{elected}$.

Compositional Structures. Our approach's core structures are *sequential*, *parallel* and *loop composition*, as well as the *revision* of decisions by prior modules. When composing two electoral modules $m \triangleright n$ sequentially, the second module n only decides on alternatives which m defers and cannot reduce the alternatives already elected or rejected. A parallel composition $m||_a n$ delegates the two set-triples of m and n to an *aggregator* a, another component type which combines two such triples into one triple. Moreover, we may revise choices from prior modules and defer them for further decisions using a revision structure $\downarrow$. Finally, a loop composition $m \circlearrowleft_t$ reiterates a module m sequentially until either m's iteration reaches a fixed point, or a *termination condition* t holds, i.e., a component type which is simply a predicate on a triple of sets of alternatives.

A Simple Example. The well-known *Baldwin's rule* [2] can be sequentially composed with a loop structure of a module eliminating the alternative with the lowest Borda score and terminating when only one alternative remains, and a module which elects all deferred alternatives. This construction directly establishes, e.g., the Pareto property and Condorcet consistency as the loop may never reject a Condorcet winner and always rejects Pareto-dominated alternatives.

3 Related Work, Conclusion and Outlook

Related Work. Our electoral modules are based on less-formal components for hierarchical electoral systems from [4]. Other work designs voting rules less modularly for statistically guaranteeing social choice properties by machine learning [7]. Prior modular approaches target verification [5] or declarative combinations of voting rules [3], but ignore social choice properties. Specific compositional structures as presented in [6] are readily expressible by our structures.

Conclusion. Our approach enables flexible and intuitive compositions of voting rules from a small number of structures with precise and general interfaces, easily extended with further modules. This allows to formally establish common social choice properties from given component properties by rigorous composition rules.

Outlook. A formally verified application of our approach is underway.

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